

SRW computation

Implementation of the computer-assisted proof of the paper “Generalized approach to the non-backtracking lace expansion” by Robert Fitzner and Remco van der Hofstad

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Abstract

This document is the first part of the computer-assisted proof of the non-backtracking lace expansion (NoBLE). The NoBLE was created by Remco van der Hofstad and the author to prove mean-field behavior for self-avoiding walk (SAW), lattice tree (LT), lattice animals (LA) and percolation. In this file the model-independent computations, explain in “Generalized approach to the non-backtracking lace expansion” Section 4, are computed. All reference in this file are to this paper.

We compute the Green’s function of simple random walk (SRW) and other related quantities. The computation of SRW-integrals are based upon techniques of Takashi Hara and Gordon Slade “The lace expansion for self-avoiding walk in five or more dimensions.” (1991). Additionally, we store the number of self-avoiding and bond-avoiding walks that are used in the model-dependent files.

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What to do with this file.

You use this file, together with the model-dependent file, to perform the computer-assisted part of the NoBLE. If succesful we prove the infrared bound for a given model in a given dimension. The only input that is required is the dimension, see just below. Once selected you should compile the hole file and can then proceed with the model dependent file. You only need to return to this file if you want to change the dimensions that you consider.

Input

The dimension in which we perfrom the computations

```
d = 15;  
param = 4; (*param=3, SAW if (d≥ 8), param=4 percolation (d≥ 15),  
param=5 LT and LA and if d≥ 17*)
```

We compute the SRW-Bubbles, SRW-triangle, SRW-square and SRW-pentagon, and prescribe a precision of 20 digits. While the SRW-pentagon is finite for dimensions $d \geq 11$, *Mathematica* guarantees the prescribed precision only for $d \geq 17$. This is sufficient for LT and LA as the NoBLE works for these models only in $d \geq 20$. For percolation and SAW we do not require the SRW-pentagons. For param=4 the pentagon are not computed and the desired precision is garanteed by *Mathematica* for $d \geq 15$. For param=3 we also do not compute squares and can perform the computation for $d \geq 8$.

If you want to consider smaller dimensions or want to consider the techniques explain in the Appendix of the model-dependent papers, please use the advanced version of this file.

For the reader of this file

This file consists of three Sections:

- In Section 1, we compute the SRW integral $I_{n,l}$, which corresponds to the SRW Green’s function.

- In Section 2, we compute bounds on other SRW-integrals, using the value of $I_{n,l}$.
- In Section 3, we store the number of self-avoiding and bond-avoiding walks.

In this basic version of the file the computations are done via build-in function of *Mathematica* or via explicite computation. This should also allows readers with only minor experience in programming to understand this file.

Next, we highlight a feature of the implementation. Namely, the coordinates which we use to identify points $x \in \mathbb{Z}^d$. We map the standard representation of a vector $x = (x_1, x_2, \dots, x_d)$ as to

$$x \mapsto v(x) = (N_1(x), N_2(x), N_3(x), N_4(x), \dots)$$

where $N_j(x) =$ number of x_i with $|x_i| = j$. For example we represent x as follows

$$\begin{aligned} x = e_1 &\mapsto v(x) = \{1, 0, 0, 0, \dots\} \\ x = 4e_1 &\mapsto v(x) = \{0, 0, 0, 1, \dots\} \\ x = -4e_5 &\mapsto v(x) = \{0, 0, 0, 1, \dots\} \\ x = e_1 + e_2 + 2e_2 + 3e_4 &\mapsto v(x) = \{2, 1, 1, 0, \dots\} \end{aligned}$$

This mapping is not a bijection. However, it incodes all necessary informations as the considered SRW quantities have various symmetries, e.g. $I_{n,l}(2e_1 + e_2) = I_{n,l}(e_3 - 2e_5)$. Thus, this description via $v(x)$ carries all the information required to compute the value of a SRW-integral. We are only interested in x that are close to the origin, and we end our representation $v(x)$ after the last non-trivial entry, i.e. $v(x) = \{0, 1, 1, 0, \dots\} = \{0, 1, 1\}$.

1. Simple Random Walk integral $I_{n,l}(x)$

We compute the two-point function of the simple random walk,

$$I_{n,l}(x) = \int_{[-\pi, \pi]^d} e^{ik \cdot x} \frac{\hat{D}^l(k)}{(1 - \hat{D}(k))^n} \frac{d^d k}{(2\pi)^d}.$$

We use the following relations

$$I_{n,l}(x) = I_{n,(l-1)}(x) - I_{(n-1),(l-1)}(x).$$

to reduce the computation of $I_{n,l}(x)$ to the computation of $I_{0,l}(x)$ and $I_{n,0}(x)$. We split this section into four part:

- select the points x for which we want to compute $I_{n,l}(x)$
- compute $I_{0,l}(x)$
- compute $I_{n,0}(x)$
- use the relation above to compute $I_{n,l}(x)$

1.1 Selection of the x

To compute the bounds on the NoBLE coefficients, as explain the main part of the model dependent articles, we require the values of $I_{n,l}(x)$, $L_n(x)$ CITE for the following values of x :

```
NvecZero = {0}; (* x=0 *)
Nveci = {1}; (* x= e1 *)
Nvecik = {2}; (* x= e1+e2 *)
NvecTwoi = {0, 1};
(* x=2 e1 *)
```

To compute $L_n(x)$ for these values we also need to compute $I_{n,l}(x)$ for the following values (see also CITE) :

```

NvecThreei = {0, 0, 1}; (* x=3 e1 *)
NvecFouri = {0, 0, 0, 1}; (* x=4 e1 *)
NvecTwoiTwok = {0, 2}; (* x=2 e1+2 e2 *)
NvecTwoik = {1, 1}; (* x=2 e1+e2 *)
NvecThreeik = {1, 0, 1}; (* x=3 e1+e2 *)
NvecTwoikp = {2, 1}; (* x=2 e1+e2+e3 *)
Nvecikpj = {4}; (* x= e1+e2+e3+e4 *)
NVecAll = {NvecZero, Nveci, NvecTwoi, NvecThreei, NvecFouri, Nvecik,
  NvecTwoikp, NvecTwoiTwok, NvecTwoik, NvecThreeik, Nvecikpj};

```

1.2 Computation of $I_{o,l}(x)$

Reviewing the definition it is easy to see that $(2d)^l I_{o,l}(x) = p_l(x)$, where $p_l(x)$ is the number of l -step simple random walk ending at x . We can compute the value of $p_l(x)$ using simple combinatorics, see Section 4.1.2. We did the combinatorics manually and save the data in the following:

```

MaxNumberOfSteps = 12;
Do[Do[NumberOfSRWTox[v, m] = 0, {m, 0, 12}], {v, NVecAll}]
(* p_n(0) *)
NumberOfSRWTox[{0}, 0] = 1;
NumberOfSRWTox[{0}, 2] = N[2 d];
NumberOfSRWTox[{0}, 4] = N[ $d * \frac{4!}{2 \times 2} + \frac{d(d-1)}{2} * 4!$ ];
NumberOfSRWTox[{0}, 6] = N[ $d * \frac{6!}{3! 3!} + d * (d-1) * \frac{6!}{2 \times 2} + \frac{d(d-1)(d-2)}{3!} * 6!$ ];
NumberOfSRWTox[{0}, 8] =
  N[ $\left( d * \frac{8!}{4! 4!} + d * (d-1) * \left( \frac{8!}{3! 3!} + \frac{8!}{2^5} \right) + \frac{d(d-1)(d-2)}{2} * \frac{8!}{2 \times 2} + \frac{d(d-1)(d-2)(d-3)}{4!} * 8! \right)$ ];
NumberOfSRWTox[{0}, 10] =
  N[ $\left( d * \frac{10!}{5! 5!} + d * (d-1) * \left( \frac{10!}{4! 4!} + \frac{10!}{3! 3! 2! 2!} \right) + \frac{d(d-1)(d-2)}{2} \left( \frac{10!}{3! \times 3!} + \frac{10!}{(2!)^4} \right) + \frac{d(d-1)(d-2)(d-3)}{3!} * \frac{10!}{2! 2!} + \frac{d(d-1)(d-2)(d-3)(d-4)}{5!} * 10! \right)$ ];
NumberOfSRWTox[{0}, 12] =
  N[ $\left( d * \frac{12!}{6! 6!} + d * (d-1) * 12! \left( \frac{1}{5! 5!} + \frac{1}{4! 4! 2! 2!} + \frac{1}{(3!)^4 2!} \right) + \frac{d(d-1)(d-2) 12!}{4! \times 4! 2!} \left( \frac{1}{(3!)^2 (2!)^2} + \frac{1}{(2!)^6 3!} \right) + \frac{d(d-1)(d-2)(d-3) 12!}{3! 3! 3!} \left( \frac{1}{(2!)^4 2! 2!} \right) + \frac{d(d-1)(d-2)(d-3)(d-4) 12!}{2! 2! 4!} + 12! * \frac{1}{6!} d(d-1)(d-2)(d-3)(d-4)(d-5) \right)$ ];

```

(* p_n(e₁) *)

$$\text{Do}[\text{NumberOfSRWTox}[\{1\}, m] = \frac{1}{2d} \text{NumberOfSRWTox}[\{0\}, m+1], \{m, \{1, 3, 5, 7, 9, 11\}\}]$$

(* p_n(2e₁) *)

$$\text{NumberOfSRWTox}[\{0, 1\}, 2] = 1;$$

$$\text{NumberOfSRWTox}[\{0, 1\}, 4] = \frac{4!}{3!} + (d-1) \frac{4!}{2!};$$

$$\text{NumberOfSRWTox}[\{0, 1\}, 6] = \frac{6!}{4! 2!} + (d-1) 6! \left(\frac{1}{(2!)^3} + \frac{1}{3!} \right) + \frac{(d-1)(d-2) 6!}{2 2!};$$

$$\begin{aligned} \text{NumberOfSRWTox}[\{0, 1\}, 8] &= \frac{8!}{5! 3!} + (d-1) 8! \left(\frac{1}{3! 3! 2!} + \frac{1}{2! 2! 3!} + \frac{1}{4! 2!} \right) + \\ &(d-1)(d-2) 8! \left(\frac{1}{(2!)^3} + \frac{1}{2 \times 3!} \right) + \frac{(d-1)(d-2)(d-3) 8!}{3! 2!}; \end{aligned}$$

$$\text{NumberOfSRWTox}[\{0, 1\}, 10] =$$

$$\frac{10!}{6! 4!} + (d-1) 10! \left(\frac{1}{4! 4! 2!} + \frac{1}{3! 3! 3!} + \frac{1}{2! 2! 4! 2!} + \frac{1}{5! 3!} \right) +$$

$$(d-1)(d-2) 10! \left(\frac{1}{3! 3! 2!} + \frac{1}{2! 2!} \left(\frac{1}{(2!)^3 2} + \frac{1}{3!} \right) + \frac{1}{4! 2! 2} \right) +$$

$$(d-1)(d-2)(d-3) 10! \left(\frac{1}{(2!)^3 2!} + \frac{1}{3! 3!} \right) + \frac{(d-1)(d-2)(d-3)(d-4) 10!}{4! 2!};$$

$$\text{NumberOfSRWTox}[\{0, 1\}, 12] =$$

$$\frac{12!}{7! 5!} +$$

$$(d-1) 12! \left(\frac{1}{5! 5! 2!} + \frac{1}{4! 4! 3!} + \frac{1}{3! 3! 4! 2!} + \frac{1}{2! 2! 5! 3!} + \frac{1}{6! 4!} \right) +$$

$$(d-1)(d-2) 12!$$

$$\left(\frac{1}{4! 4! 2!} + \frac{1}{3! 3!} \left(\frac{1}{2! 2! 2!} + \frac{1}{3!} \right) + \frac{1}{2! 2!} \left(\frac{1}{(2!)^3 3!} + \frac{1}{4! 2!} \right) + \frac{1}{2! 5! 3!} \right) +$$

$$(d-1)(d-2)(d-3) 12! \left(\frac{1}{3! 3! 2! 2!} + \frac{1}{2! 2!} \left(\frac{1}{(2!)^3 2!} + \frac{1}{3! 2!} \right) + \frac{1}{3! 4! 2!} \right) +$$

$$(d-1)(d-2)(d-3)(d-4) 12! \left(\frac{1}{2! 2! 3! 2!} + \frac{1}{4! 3!} \right) +$$

$$\frac{1}{5!} (d-1)(d-2)(d-3)(d-4)(d-5) \frac{12!}{2!};$$

(* p_n(3e₁) *)

$$\text{NumberOfSRWTox}[\{0, 0, 1\}, 3] = 1;$$

$$\text{NumberOfSRWTox}[\{0, 0, 1\}, 5] = \frac{5!}{4!} + (d-1) \frac{5!}{3!};$$

$$\text{NumberOfSRWTox}[\{0, 0, 1\}, 7] =$$

$$\frac{7!}{5! 2!} + (d-1) \left(\frac{7!}{2! 2! 3!} + \frac{7!}{4!} \right) + (d-1)(d-2) \frac{7!}{3! 2!};$$

$$\text{NumberOfSRWTox}[\{0, 0, 1\}, 9] =$$

$$\frac{9!}{6! 3!} + (d-1) \left(\frac{9!}{3! 3! 3!} + \frac{9!}{2! 2! 4! 1!} + \frac{9!}{1! 1! 5! 2!} \right) +$$

$$(d-1)(d-2) \left(\frac{9!}{2! 2! 3!} + \frac{9!}{4! 2!} \right) + (d-1)(d-2) \frac{(d-3) 9!}{3! 3!};$$

$$\text{NumberOfSRWTox}[\{0, 0, 1\}, 11] =$$

$$\frac{11!}{7! 4!} + (d-1) 11! \left(\frac{1}{4! 4! 3!} + \frac{1}{3! 3! 4! 1!} + \frac{1}{2! 2! 5! 2!} + \frac{1}{1! 1! 6! 3!} \right) +$$

$$(d-1)(d-2)11! \left(\frac{1}{3!3!3!} + \frac{1}{2!2!} \left(\frac{1}{2!2!2!3!} + \frac{1}{4!} \right) + \frac{1}{2!5!2!} \right) +$$

$$(d-1)(d-2)(d-3)11! \left(\frac{1}{2!2!2!3!} + \frac{1}{3!4!} \right) + (d-1)(d-2)(d-3) \frac{(d-4)11!}{4!3!};$$

(* p_n(4e_1) *)

$$\text{NumberOfSRWTox}[\{0, 0, 0, 1\}, 4] = 1;$$

$$\text{NumberOfSRWTox}[\{0, 0, 0, 1\}, 6] = \frac{6!}{5!1!} + (d-1) \frac{6!}{4!};$$

$$\text{NumberOfSRWTox}[\{0, 0, 0, 1\}, 8] =$$

$$\frac{8!}{6!2!} + (d-1)8! \left(\frac{1}{(2!)^2 4!} + \frac{1}{5!} \right) + \frac{(d-1)(d-2)8!}{2 \cdot 4!};$$

$$\text{NumberOfSRWTox}[\{0, 0, 0, 1\}, 10] =$$

$$\frac{10!}{7!3!} + (d-1)10! \left(\frac{1}{3!3!4!} + \frac{1}{2!2!5!} + \frac{1}{6!2!} \right) +$$

$$(d-1)(d-2)10! \left(\frac{1}{(2!)^2 4!} + \frac{1}{2 \times 5!} \right) + \frac{(d-1)(d-2)(d-3)10!}{3!4!};$$

$$\text{NumberOfSRWTox}[\{0, 0, 0, 1\}, 12] =$$

$$\frac{12!}{8!4!} + (d-1)12! \left(\frac{1}{4!4!4!} + \frac{1}{3!3!5!} + \frac{1}{2!2!6!2!} + \frac{1}{7!3!} \right) +$$

$$(d-1)(d-2)12! \left(\frac{1}{(3!)^2 4!} + \frac{1}{(2!)^5 4!} + \frac{1}{2!2!5!} + \frac{1}{2 \times 6!2!} \right) +$$

$$(d-1)(d-2)(d-3)12! \left(\frac{1}{2!2!2!4!} + \frac{1}{3!5!} \right) + \frac{(d-1)(d-2)(d-3)(d-4)12!}{4!4!};$$

(* p_n(e_1+e_2) *)

$$\text{NumberOfSRWTox}[\{2\}, 2] = 2;$$

$$\text{NumberOfSRWTox}[\{2\}, 4] = 2 * \frac{4!}{2!} + (d-2)4!;$$

$$\text{NumberOfSRWTox}[\{2\}, 6] = 6! \left(\frac{2}{3!2!} + \frac{1}{(2!)^2} \right) + (d-2)6! \left(\frac{1}{2!2!} + \frac{2}{2!} \right) +$$

$$(d-2) \frac{(d-3)}{2} 6!;$$

$$\text{NumberOfSRWTox}[\{2\}, 8] = 2 * 8! \left(\frac{1}{4!3!} + \frac{1}{3!2!2!1!} \right) +$$

$$(d-2)8! \left(\frac{1}{3!3!} + \frac{2}{2!2!2!1!1!} + \frac{1}{(2!1!)^2} + \frac{2}{3!2!} \right) +$$

$$(d-2)(d-3)8! \left(\frac{1}{2!2!} + \frac{2}{2!2!} \right) + \frac{(d-2)(d-3)(d-4)}{3!} 8!;$$

$$\text{NumberOfSRWTox}[\{2\}, 10] = 2 * 10! \left(\frac{1}{5!4!} + \frac{1}{4!3!2!} + \frac{1}{(3!2!)^2 2!} \right) +$$

$$(d-2)10! \left(\frac{1}{4!4!} + \frac{2}{3!3!2!1!1!} + \frac{1}{2!2!2!2!} + \frac{2}{2!2!3!2!} + \right.$$

$$\left. \frac{2}{4!3!} + \frac{2}{3!2!2!} \right) +$$

$$(d-2)(d-3)10! \left(\frac{1}{3!3!} + \frac{1}{(2!)^5} + \frac{2}{2!2!2!} + \frac{2}{2!3!2!} + \frac{1}{2!2!2!} \right) +$$

$$(d-2)(d-3)(d-4)10! \left(\frac{1}{2!2!2!} + \frac{2}{3!2!} \right) + (d-2)(d-3)(d-4)(d-5) \frac{10!}{4!};$$

$$\text{NumberOfSRWTox}[\{2\}, 12] = 2 * 12! \left(\frac{1}{6!5!} + \frac{1}{5!4!2!} + \frac{1}{4!3!3!2!} \right) +$$

$$\begin{aligned}
& (d-2) 12! \left(\frac{1}{5! 5!} + \frac{2}{4! 4! 2!} + \frac{2}{3! 3! 3! 2!} + \frac{1}{3! 3! 2! 2!} + \right. \\
& \quad \left. \frac{2}{2! 2! 4! 3!} + \frac{2}{2! 2! 2! 3! 2!} + \frac{2}{5! 4!} + \frac{2}{4! 3! 2!} + \frac{1}{3! 2! 3! 2!} \right) + \\
& (d-2) (d-3) 12! \\
& \left(\frac{1}{4! 4!} + \frac{1}{3! 3! 2! 2!} + \frac{2}{3! 3! 2!} + \frac{2}{(2!)^5 2!} + \frac{1}{(2!)^4} + \frac{2}{2! 2! 3! 2!} + \right. \\
& \quad \left. \frac{2}{4! 3! 2!} + \frac{2}{3! 2! 2! 2!} \right) + \\
& (d-2) (d-3) (d-4) 12! \left(\frac{1}{3! 3! 2!} + \frac{1}{(2!)^5} + \frac{2}{(2!)^4} + \frac{2}{3! 2! 3!} + \frac{1}{3! 2! 2!} \right) + \\
& (d-2) (d-3) (d-4) (d-5) 12! \left(\frac{1}{2! 2! 3!} + \frac{2}{4! 2!} \right) + \\
& (d-2) (d-3) (d-4) (d-5) (d-6) \frac{12!}{5!};
\end{aligned}$$

(* p_n(2e1+2e2) *)

$$\text{NumberOfSRWTox}[\{0, 2\}, 4] = \frac{4!}{2! 2!};$$

$$\text{NumberOfSRWTox}[\{0, 2\}, 6] = 2 \frac{6!}{2! 3!} + (d-2) \frac{6!}{2! 2!};$$

$$\text{NumberOfSRWTox}[\{0, 2\}, 8] =$$

$$\left(\frac{8!}{3! 3!} + \frac{2 \times 8!}{4! 2! 2!} \right) + (d-2) 8! \left(\frac{1}{2! 2! 2! 2!} + \frac{2}{3! 2!} \right) + \frac{(d-2) (d-3)}{2} \frac{8!}{2! 2!};$$

$$\text{NumberOfSRWTox}[\{0, 2\}, 10] = 10! \left(\frac{2}{5! 3! 2!} + \frac{2}{4! 2! 3! 1!} \right) +$$

$$(d-2) 10! \left(\frac{2}{4! 2! 2!} + \frac{2}{2! 2! 3! 2!} + \frac{1}{3! 3!} \right) +$$

$$(d-2) (d-3) 10! \left(\frac{1}{(2!)^4} + \frac{2}{3! 2! 2!} \right) + (d-2) (d-3) \frac{(d-4)}{3!} \frac{10!}{2! 2!};$$

$$\text{NumberOfSRWTox}[\{0, 2\}, 12] = 12! \left(\frac{2}{5! 5! 2!} + \frac{2}{4! 4! 3!} + \frac{1}{(4! 2! 2!)} \right) +$$

$$(d-2) 12! \left(\frac{1}{4! 4! 2! 2!} + \frac{2}{3! 3! 3! 2!} + \frac{2}{2! 2! 4! 2!} + \frac{1}{2! 2! 3! 3!} + \right. \\
\left. \frac{2}{5! 3! 2!} + \frac{2}{4! 2! 3!} \right) +$$

$$(d-2) (d-3) 12!$$

$$\left(\frac{1}{2! 2! 3! 2!} + \frac{1}{(2!)^5 2! 2!} + \frac{1}{2! 2! 3! 2!} + \frac{2}{4! 2! 2!} + \frac{1}{3! 3!} \right) +$$

$$(d-2) (d-3) (d-4) 12! \left(\frac{2}{3! 2! 3!} + \frac{1}{(2!)^5} \right) + (d-2) (d-3) (d-4) \frac{(d-5)}{4!} \frac{12!}{2! 2!};$$

(* p_n(2e1+e2) *)

$$\text{NumberOfSRWTox}[\{1, 1\}, 3] = \frac{3!}{2!};$$

$$\text{NumberOfSRWTox}[\{1, 1\}, 5] = \frac{5!}{3!} + \frac{5!}{2! 2!} + (d-2) \frac{5!}{2!};$$

$$\text{NumberOfSRWTox}[\{1, 1\}, 7] =$$

$$\begin{aligned}
& 7! \left(\frac{1}{4! 2!} + \frac{1}{3! 2!} + \frac{1}{2! 3! 2!} \right) + (d-2) 7! \left(\frac{1}{2! 2! 2!} + \frac{1}{3!} + \frac{1}{2! 2!} \right) + \\
& \frac{(d-2) (d-3)}{2} \frac{7!}{2!};
\end{aligned}$$

$$\begin{aligned} \text{NumberOfSRWTox}[\{1, 1\}, 9] &= 9! \left(\frac{1}{5! 3!} + \frac{1}{4! 2! 2!} + \frac{1}{3! 3! 2!} + \frac{1}{2! 4! 3!} \right) + \\ & (d-2) 9! \left(\frac{1}{3! 3! 2!} + \frac{1}{2! 2! 3!} + \frac{1}{2! 2! 2! 2!} + \frac{1}{4! 2!} + \frac{1}{3! 2!} + \frac{1}{2! 3! 2!} \right) + \\ & (d-2) (d-3) 9! \left(\frac{1}{2! 2! 2!} + \frac{1}{3! 2!} + \frac{1}{2! 2! 2!} \right) + \frac{(d-2) (d-3) (d-4) 9!}{3! 2!}; \end{aligned}$$

$$\text{NumberOfSRWTox}[\{1, 1\}, 11] =$$

$$\begin{aligned} & 11! \left(\frac{1}{6! 4!} + \frac{1}{5! 3! 2!} + \frac{1}{4! 2! 3! 2!} + \frac{1}{3! 4! 3!} + \frac{1}{2! 5! 4!} \right) + \\ & (d-2) 11! \\ & \left(\frac{1}{4! 4! 2!} + \frac{1}{3! 3!} \left(\frac{1}{3!} + \frac{1}{2! 2!} \right) + \frac{1}{2! 2!} \left(\frac{1}{4! 2!} + \frac{1}{3! 2!} + \frac{1}{2! 3! 2!} \right) + \right. \\ & \left. \frac{1}{5! 3!} + \frac{1}{4! 2! 2!} + \frac{1}{3! 3! 2!} + \frac{1}{2! 4! 3!} \right) + \\ & (d-2) (d-3) 11! \\ & \left(\frac{1}{3! 3! 2!} + \frac{1}{(2!)^5 2!} + \frac{1}{2! 2! 3!} + \frac{1}{2! 2! 2! 2!} + \frac{1}{2! 4! 2!} + \right. \\ & \left. \frac{1}{2! 3! 2!} + \frac{1}{2! 2! 3! 2!} \right) + \\ & (d-2) (d-3) (d-4) 11! \left(\frac{1}{(2!)^4} + \frac{1}{3! 3!} + \frac{1}{3! 2! 2!} \right) + \\ & \frac{(d-2) (d-3) (d-4) (d-5) 11!}{4! 2!}; \end{aligned}$$

(* p_n(3e1+e2) *)

$$\text{NumberOfSRWTox}[\{1, 0, 1\}, 4] = \frac{4!}{3!};$$

$$\text{NumberOfSRWTox}[\{1, 0, 1\}, 6] = 6! \left(\frac{1}{4!} + \frac{1}{3! 2!} \right) + (d-2) \frac{6!}{3!};$$

$$\text{NumberOfSRWTox}[\{1, 0, 1\}, 8] =$$

$$\begin{aligned} & 8! \left(\frac{1}{5! 2!} + \frac{1}{4! 1! 2!} + \frac{1}{3! 3! 2!} \right) + (d-2) 8! \left(\frac{1}{2! 2! 3!} + \frac{1}{4!} + \frac{1}{3! 2!} \right) + \\ & \frac{(d-2) (d-3) 8!}{2 3!}; \end{aligned}$$

$$\text{NumberOfSRWTox}[\{1, 0, 1\}, 10] =$$

$$\begin{aligned} & 10! \left(\frac{1}{6! 3!} + \frac{1}{5! 2! 2!} + \frac{1}{4! 3! 2!} + \frac{1}{3! 4! 3!} \right) + \\ & (d-2) 10! \left(\frac{1}{3! 3! 3!} + \frac{1}{2! 2! 4!} + \frac{1}{2! 2! 3! 2!} + \frac{1}{5! 2!} + \frac{1}{4! 2!} + \right. \\ & \left. \frac{1}{3! 3! 2!} \right) + (d-2) (d-3) 10! \left(\frac{1}{2! 2! 3!} + \frac{1}{4! 2!} + \frac{1}{3! 2! 2!} \right) + \\ & (d-2) (d-3) \frac{(d-4) 10!}{3! 3!}; \end{aligned}$$

$$\text{NumberOfSRWTox}[\{1, 0, 1\}, 12] =$$

$$\begin{aligned} & 12! \left(\frac{1}{7! 4!} + \frac{1}{6! 3! 2!} + \frac{1}{5! 2! 3! 2!} + \frac{1}{4! 1! 4! 3!} + \frac{1}{3! 5! 4!} \right) + \\ & (d-2) 12! \\ & \left(\frac{1}{4! 4! 3!} + \frac{1}{3! 3!} \left(\frac{1}{4!} + \frac{1}{3! 2!} \right) + \frac{1}{2! 2!} \left(\frac{1}{5! 2!} + \frac{1}{4! 2!} + \frac{1}{3! 3! 2!} \right) + \right. \end{aligned}$$

$$\frac{1}{6! 3!} + \frac{1}{5! 2! 2!} + \frac{1}{4! 3! 2!} + \frac{1}{3! 4! 3!} \Big) +$$

$$(d-2) (d-3) 12!$$

$$\left(\frac{1}{3! 3! 3!} + \frac{1}{(2!)^5 3!} + \frac{1}{2! 2!} \left(\frac{1}{4!} + \frac{1}{3! 2!} \right) + \frac{1}{2!} \left(\frac{1}{5! 2!} + \frac{1}{4! 2!} + \frac{1}{3! 3! 2!} \right) \right) +$$

$$(d-2) (d-3) (d-4) 12! \left(\frac{1}{(2!)^3 3!} + \frac{1}{3! 4!} + \frac{1}{3! 3! 2!} \right) +$$

$$\frac{(d-2) (d-3) (d-4) (d-5) 12!}{4! 3!};$$

(* p_n(2e_1+e_2+e_3) *)

$$\text{NumberOfSRWTox}[\{2, 1\}, 4] = \frac{4!}{2!};$$

$$\text{NumberOfSRWTox}[\{2, 1\}, 6] = 6! \left(\frac{2}{2! 2!} + \frac{1}{3!} \right) + (d-3) \frac{6!}{2!};$$

$$\text{NumberOfSRWTox}[\{2, 1\}, 8] =$$

$$8! \left(\frac{2}{3! 2! 2!} + \frac{1}{2! 2! 2!} + \frac{2}{2! 3!} + \frac{1}{4! 2!} \right) + (d-3) 8! \left(\frac{2}{2! 2!} + \frac{1}{3!} + \frac{1}{2! 2! 2!} \right) +$$

$$\frac{(d-3) (d-4) 8!}{2 2!};$$

$$\text{NumberOfSRWTox}[\{2, 1\}, 10] =$$

$$10! \left(\frac{2}{4! 3! 2!} + \frac{2}{3! 2! 2! 2!} + \frac{2}{3! 3! 2!} + \frac{1}{2! 2! 3!} + \frac{2}{2! 4! 2!} + \frac{1}{5! 3!} \right) +$$

$$(d-3) 10! \left(\frac{2}{3! 2! 2!} + \frac{1}{2! 2! 2!} + \frac{2}{2! 3!} + \frac{2}{2! 2! 2! 2!} + \frac{1}{4! 2!} + \right.$$

$$\left. \frac{1}{3! 2! 2!} + \frac{1}{2! 3! 3!} \right) + (d-3) (d-4) 10! \left(\frac{2}{2! 2! 2!} + \frac{1}{3! 2!} + \frac{1}{2! 2! 2!} \right) +$$

$$\frac{(d-3) (d-4) (d-5) 10!}{3! 2!};$$

$$\text{NumberOfSRWTox}[\{2, 1\}, 12] =$$

$$12! \left(\frac{1}{6! 4!} + \frac{2}{5! 3! 2!} + \frac{2}{4! 2! 3! 2!} + \frac{1}{4! 2! 2! 2!} + \frac{2}{3! 4! 3!} + \right.$$

$$\left. \frac{2}{3! 3! 2! 2!} + \frac{2}{2! 5! 4!} + \frac{2}{2! 4! 3! 2!} + \frac{1}{2! (3! 2!)^2} \right) +$$

$$(d-3) 12! \left(\frac{1}{4! 4! 2!} + \frac{1}{3! 3! 3!} + \frac{2}{3! 3! 2! 2!} + \frac{1}{2! 2! 4! 2!} + \right.$$

$$\left. \frac{2}{2! 2! 3! 2!} + \frac{2}{(2!)^3 3! 2!} + \frac{1}{(2!)^5} + \frac{1}{5! 3!} + \frac{2}{4! 2! 2!} + \frac{2}{3! 3! 2!} + \right.$$

$$\left. \frac{1}{3! 2! 2!} + \frac{2}{2! 4! 3!} + \frac{2}{2! 3! 2! 2!} \right) +$$

$$(d-3) (d-4) 12!$$

$$\left(\frac{1}{3! 3! 2!} + \frac{1}{2^6} + \frac{1}{2! 2! 3!} + \frac{2}{2! 2! 2! 2!} + \frac{1}{2! 4! 2!} + \frac{2}{2! 3! 2!} + \right.$$

$$\left. \frac{2}{2! 2! 3! 2!} + \frac{1}{2! 2! 2! 2!} \right) +$$

$$(d-3) (d-4) (d-5) 12! \left(\frac{1}{2! 2! 2! 2!} + \frac{1}{3! 3!} + \frac{2}{2! 2! 3!} \right) +$$

$$\frac{(d-3) (d-4) (d-5) (d-6) 12!}{4! 2!};$$


```

(* p_n(e1+e2+e3+e4) *)
NumberOfSRWTox[{4}, 4] = 4!;
NumberOfSRWTox[{4}, 6] = 6!  $\frac{4}{2!}$  + (d-4) 6!;
NumberOfSRWTox[{4}, 8] = 8!  $\left(\frac{4}{3! 2!} + \frac{4!}{2! 2!} \frac{1}{2! 2!}\right)$  + (d-4) 8!  $\left(\frac{4}{2!} + \frac{1}{2! 2!}\right)$  +
(d-4)  $\frac{(d-5)}{2}$  8!;
NumberOfSRWTox[{4}, 10] = 10!  $\left(\frac{4}{4! 3!} + 4 * 3 * \frac{1}{3! 2! 2!} + 4 \frac{1}{(2!)^3}\right)$  +
(d-4) 10!  $\left(\frac{4}{3! 2!} + \frac{4!}{2! 2! 2! 2!} + \frac{4}{2! 2! 2!} + \frac{1}{3! 3!}\right)$  +
(d-4) (d-5) 10!  $\left(\frac{4}{2! 2!} + \frac{1}{2! 2!}\right)$  +  $\frac{(d-4)(d-5)(d-6)}{3!}$  10!;
NumberOfSRWTox[{4}, 12] =
12!  $\left(\frac{4}{5! 4!} + 4 * 3 * \frac{1}{4! 3! 2!} + \frac{4 * 3}{2} \frac{1}{(3! 2!)^2} + 4 * \frac{3 * 2}{2} \frac{1}{3! 2! 2! 2!} + \frac{1}{(2!)^4}\right)$  +
(d-4) 12!  $\left(\frac{1}{4! 4!} + \frac{4}{3! 3! 2!} + \frac{4}{2! 2! 3! 2!} + \frac{4 * 3}{2} \frac{1}{(2!)^4} + 4 \frac{1}{4! 3!} + \right.$ 
 $\left. 4 * 3 \frac{1}{3! 2! 2!} + \frac{4}{(2!)^3}\right)$  +
(d-4) (d-5) 12!  $\left(\frac{1}{3! 3!} + \frac{1}{(2!)^5} + \frac{4}{2! 2! 2!} + \frac{4}{3! 2! 2!} + \frac{4 * 3}{2} \frac{1}{2! 2! 2!}\right)$  +
(d-4) (d-5) (d-6) 12!  $\left(\frac{1}{(2!)^3} + \frac{4}{3! 2!}\right)$  +  $\frac{(d-4)(d-5)(d-6)(d-7)}{4!}$  12!;

```

1.3 Computation of $I_{n,0}(x)$

$$I_{n,0}(x) = \frac{1}{(n-1)!} \int_0^\infty t^{n-1} \prod_{i=1}^d \int_{-\pi}^\pi e^{-t/d(1-\cos(k_i))} e^{tk_i x_i} \frac{dk}{2\pi} = \frac{1}{(n-1)!} \int_0^\infty t^{n-1} \prod_{i=1}^d F(t, d, |x_i|)$$

where $F(t,d,n)$ is the modified Besselfunction.

```

F[t_, d_, N_] := e- $\frac{t}{d}$  BesselI[N,  $\frac{t}{d}$ ];
NIntproto[n_, d_, T_] :=
1 / ((n-1)!) * NIntegrate[t^(n-1) * (F[t, d, 0])^(d), {t, 0, T},
WorkingPrecision -> 20];
NIntproto2[n_, d_, T_, vecn_] :=
1 / ((n-1)!) *
NIntegrate[t^(n-1) * (F[t, d, 0])^(d - Total[vecn]) *
Product[F[t, d, i]vecn[[i]], {i, 1, Length[vecn]}], {t, 0, T}, WorkingPrecision -> 20];
SRWInt[n_, dim_, vecn_] := If[vecn == {0}, NIntproto[n, dim,  $\infty$ ],
NIntproto2[n, dim,  $\infty$ , vecn]];
Clear[i];

```

Remark: We have chosen the representation of the point $v(x)$, such that the above computation becomes as simple as it is now.

1.4 Computation of $I_{n,l}(x)$

Next, we compute the values of $I_{n,l}(x)$ for the different x using $I_{n,l}(x) = I_{n,(l-1)}(x) - I_{(n-1),(l-1)}(x)$

```
Do[
  tabletmp = Table[0, {s, 1, MaxNumberOfSteps + 2}, {t, 1, 2 + param}];
  (*creating an empty table*)
  tabletmp[[1, 1]] = Text["1 \\ n"]; (*formatting top/left entry*)
  Do[tabletmp[[1, n + 2]] = n, {n, 0, param}]; (*formatting first row of table*)
  Do[tabletmp[[1 + 2, 1]] = 1, {1, 0, MaxNumberOfSteps}];
  (*formatting first coloumn of table*)
  Do[tabletmp[[2, n + 2]] = SRWInt[n, d, v];, {n, 1, param}];
  (*compute second row*)
  Do[tabletmp[[1 + 2, 2]] =  $\frac{\text{NumberOfSRWTox}[v, 1]}{(2 d)^1}$ ;, {1, 0, MaxNumberOfSteps}];
  (*fill second coloumn*)
  Do[
    Do[tabletmp[[1 + 3, n + 2]] = tabletmp[[1 + 2, n + 2]] - tabletmp[[1 + 2, n + 1]],
      {n, 1, param}]; (*fill these rest of the table using,
     $I_{n,l}(x) = I_{n,(l-1)}(x) - I_{(n-1),(l-1)}(x)$ 
    , {1, 0, MaxNumberOfSteps - 1}];
  SRWTwoPointFunctionTable[v] = tabletmp;
  Clear[n, 1, s, t, tabletmp];
  , {v, NVecAll}]
```

1.5 Print-Out of $I_{n,l}(x)$

We print out the computed value of the SRW-Integrals. At the moment this part is deactivated to save the paper when printed.

```
(*Do[
  NForm[a_] := NumberForm[N[a], 5];
  OutputTable = Map[NForm, SRWTwoPointFunctionTable[v], {2}];
  Print[
    Labeled[Grid[OutputTable,
      Alignment -> {{Left, Center}, Baseline, {{2, 14}, {2, param+2}} -> {"."}},
      Frame -> True, Dividers -> {{2 -> True, -1 -> True}, {2 -> True}},
      Spacings -> {1.5, {1.5, 1, {0.5}}}, ItemStyle -> {1 -> Bold, 1 -> Bold},
      Background -> {Automatic, Automatic,
        {{2, 14}, {2, param+2}} -> GrayLevel[0.9]}],
    Style["Value of the SRW two-point function in dimension "Text[d]Text[v], Bold],
    Top] // Text]
  Clear[OutputTable];
  , {v, NVecAll}]*)
```

For better readability of the code we define the following function to access the values of $I_{n,l}(x)$:

```

Ivalue[n_, l_, v_] :=
Module[ {value},
  value = SRWTwoPointFunctionTable[v][[1 + 2, n + 2]];
  value
];

```

2. Other SRW-Integrals

Now, we use the computed values of $I_{n,l}(x)$ to bound the other SRW integrals, see Section CITE. We begin with $L_n(x)$ which we can compute explicitly, see CITE.

$$L_n(x)$$

```

Do[
  L[n, {1}] =  $\frac{1}{2d}$  Ivalue[n, 0, {0}] +  $\frac{1}{2d}$  Ivalue[n, 0, {0, 1}] +  $\frac{d-1}{d}$  Ivalue[n, 0, {2}];
  L[n, {0, 1}] =  $\frac{1}{2d}$  Ivalue[n, 0, {0}] +  $\frac{1}{2d}$  Ivalue[n, 0, {0, 0, 0, 1}] +
     $\frac{d-1}{d}$  Ivalue[n, 0, {0, 2}];
  L[n, {2}] =  $\frac{(d-2)(d-3)}{d(d-1)}$  Ivalue[n, 0, {4}] +
     $\frac{(d-2)}{2d(d-1)}$  (Ivalue[n, 0, {2}] + Ivalue[n, 0, {2, 1}]) +
     $\frac{1}{4d(d-1)}$  (Ivalue[n, 0, {0}] + Ivalue[n, 0, {0, 2}] + 2 Ivalue[n, 0, {0, 1}]);
, {n, 0, param}]

```

$$K_{n,l}(x)$$

Then, we bound $K_{n,l}(x)$ as described in CITE, and then improve the bounds using comment CITE

```

Do[Do[
  K[n, 1, {1}] = Min[Ivalue[n, 2 1, {0}]1/2 L[n, {1}]1/2, Ivalue[n, 1 + 1, {0}]];
  K[n, 1, {0, 1}] = Ivalue[n, 2 1, {0}]1/2 L[n, {0, 1}]1/2;
  K[n, 1, {2}] = Ivalue[n, 2 1, {0}]1/2 L[n, {2}]1/2;
  , {1, 0, 6}]
Do[
  K[n, 1, {0, 1}] = Ivalue[n, 12, {0}]1/2 L[n, {0, 1}]1/2;
  K[n, 1, {2}] = Ivalue[n, 12, {0}]1/2 L[n, {2}]1/2;
  , {1, 7, 12}]
Do[
  K[n, 1, {1}] = Min[Ivalue[n, 12, {0}]1/2 L[n, {1}]1/2, Ivalue[n, 1 + 1, {0}]];
  , {1, 7, 11}];
K[n, 12, {1}] = Min[Ivalue[n, 12, {0}]1/2 L[n, {1}]1/2, Ivalue[n, 12, {0}]];
, {n, 0, param}];

Do[Do[
  K[n, 0, v] = K[n - 1, 0, v] + Ivalue[n - 1, 4, {0}]1/2 L[n - 1, v]1/2 +
    Ivalue[n, 4, {0}]1/2 L[n, v]1/2;
  , {n, 1, param}], {v, {{1}, {0, 1}, {2}})];

```

For $x = 0$, we bound as given in CITE

```

Do[
  Do[
    K[n, 2 1, {0}] = Ivalue[n, 2 1, {0}];
    , {1, 0, 6}];
  Do[
    K[n, 2 1 + 1, {0}] =  $\sqrt{(Ivalue[n, 2 1, {0}] Ivalue[n, 2 1 + 2, {0}])}$ ;
    , {1, 0, 5}];
  , {n, 0, param}];

```

$$T_{n,t}(x)$$

We bound $T_{n,t}(x)$ as stated in CITE

```

Do[Do[Do[
  T[n, 1, v] = K[n, 1 + 1, v] + Min[ $\frac{4}{d}$  K[n, 1, v],  $\frac{2}{d}$  K[n + 1, 1, v]];
  , {n, 1, param - 1}];
  T[param, 1, v] = K[param, 1 + 1, v] +  $\frac{4}{d}$  K[param, 1, v];
  , {1, 0, 12}], {v, {{0}, {1}, {0, 1}, {2}})];

```

$$V_{n,t}$$

We compute the bounds on $V_{n,t}$ using CITE

```

Do[Do[
  V[n, 1] =
    
$$\frac{1}{(2d)^2} \left( \text{Ivalue}[n, 1, \{0\}] - 2 \text{Ivalue}[n, 1, \{0, 1\}] + \frac{d-1}{d} \text{Ivalue}[n, 1, \{0, 2\}] + \frac{1}{2d} \text{Ivalue}[n, 1, \{0\}] + \frac{1}{2d} \text{Ivalue}[n, 1, \{0, 0, 0, 1\}] \right);$$

  , {n, 0, param}]
, {1, 0, 12}];

```

$$U_{n,l}(x)$$

Then we compute $U_{n,l}(x)$ as given in CITE and CITE

```

Do[Do[Do[
  U[n, 1, v] = V[n, 2 1]^{1/2} L[n, v]^{1/2};
  , {v, {{0, 1}, {2}}}]];
If[Mod[1, 2] == 0, U[n, 1, {0}] =  $\frac{1}{2d} (\text{Ivalue}[n, 1, \{0\}] - \text{Ivalue}[n, 1, \{0, 1\}])$ ,
  U[n, 1, {0}] =  $\frac{1}{d} \text{K}[n, 1, \{0\}]$ ];
, {n, 0, param}], {1, 0, 6}];

```

```

Do[Do[Do[
  U[n, 1, v] = V[n, 12]^{1/2} L[n, v]^{1/2};
  , {v, {{0, 1}, {2}}}]];
U[n, 1, {0}] = U[n, 6, {0}];
, {n, 0, param}], {1, 7, 12}];

```

```

Do[Do[
  U[n, 1, {1}] = Min[V[n, 2 1]^{1/2} L[n, {1}]^{1/2}, U[n, 1+1, {0}]];
  , {1, 0, 5}];
U[n, 6, {1}] = V[n, 12]^{1/2} L[n, {1}]^{1/2};
, {n, 0, param}];

```

Weighted SRW-diagram

We begin with the bound for the initial point, derived in Section 2.3.3, see (2.30).

```

IM[n_, l_, v_] := Ivalue[n+2, 1+1, v] -  $\frac{1}{d}$  Ivalue[n+3, 1, v] +
 $\frac{1}{2d^2}$  Switch[v, {0}, 2 d Ivalue[n+3, 1, {0, 1}], {1},
  Ivalue[n+3, 1, {1}] + Ivalue[n+3, 1, {0, 0, 1}] + (2 d - 2) Ivalue[n+3, 1, {1, 1}],
  {0, 1}, Ivalue[n+3, 1, {0}] + Ivalue[n+3, 1, {0, 0, 0, 1}] +
  (2 d - 2) Ivalue[n+3, 1, {0, 2}], {2},
  2 Ivalue[n+3, 1, {2}] + 2 Ivalue[n+3, 1, {1, 0, 1}] +
  (2 d - 4) Ivalue[n+3, 1, {2, 1}]];

```

We remark at this point again that in this basic case only the case $S=\{0\}$ and $S=Z^d/\{0\}$ can be considered.

Print-out of the used values

We print out the value that we use for our computations:

```
(*TableSup=Table[0, {s, 1, 5}, {t, 1, 4}];
n=2; l=4;
TableSup[[1, 2]]={1}; TableSup[[1, 3]]={0, 1}; TableSup[[1, 4]]={2};
TableSup[[2, 1]]=Text[Lnl];
TableSup[[2, 2]]=L[n, {1}];
TableSup[[2, 3]]=L[n, {0, 1}];
TableSup[[2, 4]]=L[n, {2}];
TableSup[[3, 1]]=Text[Knl];
TableSup[[3, 2]]=K[n, 1, {1}];
TableSup[[3, 3]]=K[n, 1, {0, 1}];
TableSup[[3, 4]]=K[n, 1, {2}];
TableSup[[4, 1]]=Text[Unl];
TableSup[[4, 2]]=U[n, 1, {1}];
TableSup[[4, 3]]=U[n, 1, {0, 1}];
TableSup[[4, 4]]=U[n, 1, {2}];
TableSup[[5, 1]]=Text[Tnl];
TableSup[[5, 2]]=T[n, 1, {1}];
TableSup[[5, 3]]=T[n, 1, {0, 1}];
TableSup[[5, 4]]=T[n, 1, {2}];
NForm[a_]:=NumberForm[N[a], 6];
Print[
  Labeled[Grid[Map[NForm, TableSup, {2}],
    Alignment -> {{Left, Center}, Baseline, {{2, 5}, {2, 4}} -> {"."}},
    Frame -> True, Dividers -> {{2 -> True, -1 -> True}, {2 -> True}},
    Spacings -> {1.5, {1.5, 1, {0.5}}}, ItemStyle -> {1 -> Bold, 1 -> Bold},
    Background -> {Automatic, Automatic, {{2, 5}, {2, 4}} -> GrayLevel[0.9]],
    Style["Bounds of the SRW two-point function "Text[d], Bold], Top] // Text]
Clear[n, l]*)
```

We define the following functions to access the value of the supremum of the functions for $x \neq 0$:

3. Number of Self-avoiding and Bond-avoiding walks

We improve our bounds on simple diagrams by explicitly extracting short contributions. For this we have computed the number of self-avoiding walks (SAW) and bond-avoiding walks (BAW) with a specific number of steps. As we use this for all models we save/store these numbers here in the model-independent part of the implementation. These values are computed using a simple JAVA program that is available on the website of the author.

```

ComputedPoints = Join[Table[{i}, {i, 0, 10}], Table[{i, 1}, {i, 0, 8}],
  Table[{i, 0, 1}, {i, 0, 7}], Table[{i, 0, 0, 1}, {i, 0, 6}],
  Table[{i, 0, 0, 0, 1}, {i, 0, 5}], Table[{i, 0, 0, 0, 0, 1}, {i, 0, 4}],
  Table[{i, 0, 0, 0, 0, 0, 1}, {i, 0, 3}], Table[{i, 0, 0, 0, 0, 0, 0, 1}, {i, 0, 2}],
  Table[{i, 0, 0, 0, 0, 0, 0, 0, 1}, {i, 0, 1}], Table[{i, 2}, {i, 0, 6}],
  Table[{i, 0, 2}, {i, 0, 4}], Table[{i, 0, 0, 2}, {i, 0, 2}], Table[{i, 3}, {i, 0, 4}],
  Table[{i, 0, 3}, {i, 0, 1}], Table[{i, 4}, {i, 0, 2}], Table[{i, 5}, {i, 0, 0}],
  Table[{i, 1, 1}, {i, 0, 5}]];

```

```

ComputedSteps = 10;

```

```

Do[Do[

```

```

  nrSAW[n, d, v] = 0;

```

```

  nrBAW[n, d, v] = 0; , {n, 0, ComputedSteps}], {v, ComputedPoints}];

```

```

(*n=1, SAW*)

```

```

nrSAW[1, d, {1}] = If[(d ≥ 1), 1, 0];

```

```

(*n=1, BAW*)

```

```

nrBAW[1, d, {1}] = If[(d ≥ 1), 1, 0];

```

```

(*n=2, SAW*)

```

```

nrSAW[2, d, {0}] = If[(d ≥ 1), 1, 0];

```

```

nrSAW[2, d, {2}] = If[(d ≥ 2), 2, 0];

```

```

(*n=2, BAW*)

```

```

nrBAW[2, d, {0}] = If[(d ≥ 1), 1, 0];

```

```

nrBAW[2, d, {2}] = If[(d ≥ 2), 2, 0];

```

```

(*n=3, SAW*)

```

```

nrSAW[3, d, {1}] = If[(d ≥ 1), 2 (d - 1), 0];

```

```

nrSAW[3, d, {0, 0, 1}] = If[(d ≥ 1), 1, 0];

```

```

nrSAW[3, d, {1, 1}] = If[(d ≥ 2), 3, 0];

```

```

nrSAW[3, d, {3}] = If[(d ≥ 3), 6, 0];

```

```

(*n=3, BAW*)

```

```

nrBAW[3, d, {1}] = If[(d ≥ 1), 2 (d - 1), 0];

```

```

nrBAW[3, d, {0, 0, 1}] = If[(d ≥ 1), 1, 0];

```

```

nrBAW[3, d, {1, 1}] = If[(d ≥ 2), 3, 0];

```

```

nrBAW[3, d, {3}] = If[(d ≥ 3), 6, 0];

```

```

(*n=4, SAW*)

```

```

nrSAW[4, d, {2}] = If[(d ≥ 2), 4 + 12 (d - 2), 0];

```

```

nrSAW[4, d, {0, 1}] = If[(d ≥ 1), 6 (d - 1), 0];

```

```

nrSAW[4, d, {0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];

```

```

nrSAW[4, d, {1, 0, 1}] = If[(d ≥ 2), 4, 0];

```

```

nrSAW[4, d, {0, 2}] = If[(d ≥ 2), 6, 0];
nrSAW[4, d, {2, 1}] = If[(d ≥ 3), 12, 0];
nrSAW[4, d, {4}] = If[(d ≥ 4), 24, 0];
(*n=4,BAW*)
nrBAW[4, d, {2}] = If[(d ≥ 2), 4 + 12 (d - 2), 0];
nrBAW[4, d, {0, 1}] = If[(d ≥ 1), 6 (d - 1), 0];
nrBAW[4, d, {0}] = If[(d ≥ 0), 4 d (d - 1), 0];
nrBAW[4, d, {0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrBAW[4, d, {1, 0, 1}] = If[(d ≥ 2), 4, 0];
nrBAW[4, d, {0, 2}] = If[(d ≥ 2), 6, 0];
nrBAW[4, d, {2, 1}] = If[(d ≥ 3), 12, 0];
nrBAW[4, d, {4}] = If[(d ≥ 4), 24, 0];

(*n=5,SAW*)
nrSAW[5, d, {3}] = If[(d ≥ 3), 54 + 72 (d - 3), 0];
nrSAW[5, d, {1, 1}] = If[(d ≥ 2), 11 + 36 (d - 2), 0];
nrSAW[5, d, {1}] = If[(d ≥ 1), 6 (d - 1) + 16 (d - 1) (d - 2), 0];
nrSAW[5, d, {0, 0, 1}] = If[(d ≥ 1), 12 (d - 1), 0];
nrSAW[5, d, {0, 0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrSAW[5, d, {1, 0, 0, 1}] = If[(d ≥ 2), 5, 0];
nrSAW[5, d, {0, 1, 1}] = If[(d ≥ 2), 10, 0];
nrSAW[5, d, {2, 0, 1}] = If[(d ≥ 3), 20, 0];
nrSAW[5, d, {1, 2}] = If[(d ≥ 3), 30, 0];
nrSAW[5, d, {3, 1}] = If[(d ≥ 4), 60, 0];
nrSAW[5, d, {5}] = If[(d ≥ 5), 120, 0];
(*n=5,BAW*)
nrBAW[5, d, {3}] = If[(d ≥ 3), 54 + 72 (d - 3), 0];
nrBAW[5, d, {1, 1}] = If[(d ≥ 2), 11 + 36 (d - 2), 0];
nrBAW[5, d, {1}] = If[(d ≥ 1), 14 (d - 1) + 24 (d - 1) (d - 2), 0];
nrBAW[5, d, {0, 0, 1}] = If[(d ≥ 1), 12 (d - 1), 0];
nrBAW[5, d, {0, 0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrBAW[5, d, {1, 0, 0, 1}] = If[(d ≥ 2), 5, 0];
nrBAW[5, d, {0, 1, 1}] = If[(d ≥ 2), 10, 0];
nrBAW[5, d, {2, 0, 1}] = If[(d ≥ 3), 20, 0];
nrBAW[5, d, {1, 2}] = If[(d ≥ 3), 30, 0];
nrBAW[5, d, {3, 1}] = If[(d ≥ 4), 60, 0];
nrBAW[5, d, {5}] = If[(d ≥ 5), 120, 0];

(*n=6,SAW*)
nrSAW[6, d, {4}] = If[(d ≥ 4), 576 + 480 (d - 4), 0];
nrSAW[6, d, {2, 1}] = If[(d ≥ 3), 168 + 240 (d - 3), 0];
nrSAW[6, d, {2}] = If[(d ≥ 2), 16 + 168 (d - 2) + 144 (d - 2) (d - 3), 0];
nrSAW[6, d, {1, 0, 1}] = If[(d ≥ 2), 26 + 80 (d - 2), 0];
nrSAW[6, d, {0, 2}] = If[(d ≥ 2), 24 + 120 (d - 2), 0];

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nrSAW[6, d, {0, 1}] = If[(d ≥ 1), 20 (d - 1) + 72 (d - 1) (d - 2), 0];
nrSAW[6, d, {0, 0, 0, 1}] = If[(d ≥ 1), 20 (d - 1), 0];
nrSAW[6, d, {0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrSAW[6, d, {1, 0, 0, 0, 1}] = If[(d ≥ 2), 6, 0];
nrSAW[6, d, {0, 1, 0, 1}] = If[(d ≥ 2), 15, 0];
nrSAW[6, d, {2, 0, 0, 1}] = If[(d ≥ 3), 30, 0];
nrSAW[6, d, {0, 0, 2}] = If[(d ≥ 2), 20, 0];
nrSAW[6, d, {1, 1, 1}] = If[(d ≥ 3), 60, 0];
nrSAW[6, d, {3, 0, 1}] = If[(d ≥ 4), 120, 0];
nrSAW[6, d, {0, 3}] = If[(d ≥ 3), 90, 0];
nrSAW[6, d, {2, 2}] = If[(d ≥ 4), 180, 0];
nrSAW[6, d, {4, 1}] = If[(d ≥ 5), 360, 0];
nrSAW[6, d, {6}] = If[(d ≥ 6), 720, 0];
(*n=6,BAW*)
nrBAW[6, d, {4}] = If[(d ≥ 4), 576 + 480 (d - 4), 0];
nrBAW[6, d, {2, 1}] = If[(d ≥ 3), 168 + 240 (d - 3), 0];
nrBAW[6, d, {2}] = If[(d ≥ 2), 36 + 232 (d - 2) + 168 (d - 2) (d - 3), 0];
nrBAW[6, d, {1, 0, 1}] = If[(d ≥ 2), 26 + 80 (d - 2), 0];
nrBAW[6, d, {0, 2}] = If[(d ≥ 2), 24 + 120 (d - 2), 0];
nrBAW[6, d, {0, 1}] = If[(d ≥ 1), 28 (d - 1) + 84 (d - 1) (d - 2), 0];
nrBAW[6, d, {0, 0, 0, 1}] = If[(d ≥ 1), 20 (d - 1), 0];
nrBAW[6, d, {0}] = If[(d ≥ 0), 12 d (d - 1) + 32 d (d - 1) (d - 2), 0];
nrBAW[6, d, {0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrBAW[6, d, {1, 0, 0, 0, 1}] = If[(d ≥ 2), 6, 0];
nrBAW[6, d, {0, 1, 0, 1}] = If[(d ≥ 2), 15, 0];
nrBAW[6, d, {2, 0, 0, 1}] = If[(d ≥ 3), 30, 0];
nrBAW[6, d, {0, 0, 2}] = If[(d ≥ 2), 20, 0];
nrBAW[6, d, {1, 1, 1}] = If[(d ≥ 3), 60, 0];
nrBAW[6, d, {3, 0, 1}] = If[(d ≥ 4), 120, 0];
nrBAW[6, d, {0, 3}] = If[(d ≥ 3), 90, 0];
nrBAW[6, d, {2, 2}] = If[(d ≥ 4), 180, 0];
nrBAW[6, d, {4, 1}] = If[(d ≥ 5), 360, 0];
nrBAW[6, d, {6}] = If[(d ≥ 6), 720, 0];

(*n=7,SAW*)
nrSAW[7, d, {5}] = If[(d ≥ 5), 6000 + 3600 (d - 5), 0];
nrSAW[7, d, {3, 1}] = If[(d ≥ 4), 2040 + 1800 (d - 4), 0];
nrSAW[7, d, {3}] = If[(d ≥ 3), 708 + 2520 (d - 3) + 1224 (d - 3) (d - 4), 0];
nrSAW[7, d, {2, 0, 1}] = If[(d ≥ 3), 430 + 600 (d - 3), 0];
nrSAW[7, d, {1, 2}] = If[(d ≥ 3), 540 + 900 (d - 3), 0];
nrSAW[7, d, {1, 1, 1}] = If[(d ≥ 2), 45 + 636 (d - 2) + 612 (d - 2) (d - 3), 0];
nrSAW[7, d, {1, 0, 0, 1}] = If[(d ≥ 2), 52 + 150 (d - 2), 0];
nrSAW[7, d, {0, 1, 1}] = If[(d ≥ 2), 55 + 300 (d - 2), 0];

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nrSAW[7, d, {1}] = If[(d ≥ 1), 28 (d - 1) + 248 (d - 1) (d - 2) + 216 (d - 1) (d - 2) (d - 3), 0];
nrSAW[7, d, {0, 0, 1}] = If[(d ≥ 1), 54 (d - 1) + 204 (d - 1) (d - 2), 0];
nrSAW[7, d, {0, 0, 0, 0, 1}] = If[(d ≥ 1), 30 (d - 1), 0];
nrSAW[7, d, {0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrSAW[7, d, {1, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 7, 0];
nrSAW[7, d, {0, 1, 0, 0, 1}] = If[(d ≥ 2), 21, 0];
nrSAW[7, d, {2, 0, 0, 0, 1}] = If[(d ≥ 3), 42, 0];
nrSAW[7, d, {0, 0, 1, 1}] = If[(d ≥ 2), 35, 0];
nrSAW[7, d, {1, 1, 0, 1}] = If[(d ≥ 3), 105, 0];
nrSAW[7, d, {3, 0, 0, 1}] = If[(d ≥ 4), 210, 0];
nrSAW[7, d, {1, 0, 2}] = If[(d ≥ 3), 140, 0];
nrSAW[7, d, {0, 2, 1}] = If[(d ≥ 3), 210, 0];
nrSAW[7, d, {2, 1, 1}] = If[(d ≥ 4), 420, 0];
nrSAW[7, d, {4, 0, 1}] = If[(d ≥ 5), 840, 0];
nrSAW[7, d, {1, 3}] = If[(d ≥ 4), 630, 0];
nrSAW[7, d, {3, 2}] = If[(d ≥ 5), 1260, 0];
nrSAW[7, d, {5, 1}] = If[(d ≥ 6), 2520, 0];
nrSAW[7, d, {7}] = If[(d ≥ 7), 5040, 0];
(*n=7,BAW*)
nrBAW[7, d, {5}] = If[(d ≥ 5), 6000 + 3600 (d - 5), 0];
nrBAW[7, d, {3, 1}] = If[(d ≥ 4), 2040 + 1800 (d - 4), 0];
nrBAW[7, d, {3}] = If[(d ≥ 3), 1020 + 2952 (d - 3) + 1320 (d - 3) (d - 4), 0];
nrBAW[7, d, {2, 0, 1}] = If[(d ≥ 3), 430 + 600 (d - 3), 0];
nrBAW[7, d, {1, 2}] = If[(d ≥ 3), 540 + 900 (d - 3), 0];
nrBAW[7, d, {1, 1, 1}] = If[(d ≥ 2), 77 + 756 (d - 2) + 660 (d - 2) (d - 3), 0];
nrBAW[7, d, {1, 0, 0, 1}] = If[(d ≥ 2), 52 + 150 (d - 2), 0];
nrBAW[7, d, {0, 1, 1}] = If[(d ≥ 2), 55 + 300 (d - 2), 0];
nrBAW[7, d, {1}] = If[(d ≥ 1), 70 (d - 1) + 468 (d - 1) (d - 2) + 312 (d - 1) (d - 2) (d - 3), 0];
nrBAW[7, d, {0, 0, 1}] = If[(d ≥ 1), 62 (d - 1) + 220 (d - 1) (d - 2), 0];
nrBAW[7, d, {0, 0, 0, 0, 1}] = If[(d ≥ 1), 30 (d - 1), 0];
nrBAW[7, d, {0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrBAW[7, d, {1, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 7, 0];
nrBAW[7, d, {0, 1, 0, 0, 1}] = If[(d ≥ 2), 21, 0];
nrBAW[7, d, {2, 0, 0, 0, 1}] = If[(d ≥ 3), 42, 0];
nrBAW[7, d, {0, 0, 1, 1}] = If[(d ≥ 2), 35, 0];
nrBAW[7, d, {1, 1, 0, 1}] = If[(d ≥ 3), 105, 0];
nrBAW[7, d, {3, 0, 0, 1}] = If[(d ≥ 4), 210, 0];
nrBAW[7, d, {1, 0, 2}] = If[(d ≥ 3), 140, 0];
nrBAW[7, d, {0, 2, 1}] = If[(d ≥ 3), 210, 0];
nrBAW[7, d, {2, 1, 1}] = If[(d ≥ 4), 420, 0];
nrBAW[7, d, {4, 0, 1}] = If[(d ≥ 5), 840, 0];
nrBAW[7, d, {1, 3}] = If[(d ≥ 4), 630, 0];
nrBAW[7, d, {3, 2}] = If[(d ≥ 5), 1260, 0];

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nrBAW[7, d, {5, 1}] = If[(d ≥ 6), 2520, 0];
nrBAW[7, d, {7}] = If[(d ≥ 7), 5040, 0];

(*n=8, SAW*)
nrSAW[8, d, {6}] = If[(d ≥ 6), 64800 + 30240 (d - 6), 0];
nrSAW[8, d, {4, 1}] = If[(d ≥ 5), 24000 + 15120 (d - 5), 0];
nrSAW[8, d, {4}] = If[(d ≥ 4), 16296 + 32976 (d - 4) + 11040 (d - 4) (d - 5), 0];
nrSAW[8, d, {3, 0, 1}] = If[(d ≥ 4), 5760 + 5040 (d - 4), 0];
nrSAW[8, d, {2, 2}] = If[(d ≥ 4), 7800 + 7560 (d - 4), 0];
nrSAW[8, d, {2, 1}] = If[(d ≥ 3), 2586 + 10488 (d - 3) + 5520 (d - 3) (d - 4), 0];
nrSAW[8, d, {2, 0, 0, 1}] = If[(d ≥ 3), 936 + 1260 (d - 3), 0];
nrSAW[8, d, {1, 1, 1}] = If[(d ≥ 3), 1480 + 2520 (d - 3), 0];
nrSAW[8, d, {0, 3}] = If[(d ≥ 3), 1800 + 3780 (d - 3), 0];
nrSAW[8, d, {2}] =
  If[(d ≥ 2), 76 + 2468 (d - 2) + 6192 (d - 2) (d - 3) + 2480 (d - 2) (d - 3) (d - 4), 0];
nrSAW[8, d, {1, 0, 1}] = If[(d ≥ 2), 118 + 1916 (d - 2) + 1840 (d - 2) (d - 3), 0];
nrSAW[8, d, {0, 2}] = If[(d ≥ 2), 112 + 2244 (d - 2) + 2760 (d - 2) (d - 3), 0];
nrSAW[8, d, {1, 0, 0, 0, 1}] = If[(d ≥ 2), 92 + 252 (d - 2), 0];
nrSAW[8, d, {0, 1, 0, 1}] = If[(d ≥ 2), 118 + 630 (d - 2), 0];
nrSAW[8, d, {0, 0, 2}] = If[(d ≥ 2), 120 + 840 (d - 2), 0];
nrSAW[8, d, {0, 1}] = If[(d ≥ 1), 92 (d - 1) + 1216 (d - 1) (d - 2) + 1240 (d - 1) (d - 2) (d - 3),
  0];
nrSAW[8, d, {0, 0, 0, 1}] = If[(d ≥ 1), 126 (d - 1) + 460 (d - 1) (d - 2), 0];
nrSAW[8, d, {0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 42 (d - 1), 0];
nrSAW[8, d, {0, 0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrSAW[8, d, {1, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 8, 0];
nrSAW[8, d, {0, 1, 0, 0, 0, 1}] = If[(d ≥ 2), 28, 0];
nrSAW[8, d, {2, 0, 0, 0, 0, 1}] = If[(d ≥ 3), 56, 0];
nrSAW[8, d, {0, 0, 1, 0, 1}] = If[(d ≥ 2), 56, 0];
nrSAW[8, d, {1, 1, 0, 0, 1}] = If[(d ≥ 3), 168, 0];
nrSAW[8, d, {3, 0, 0, 0, 1}] = If[(d ≥ 4), 336, 0];
nrSAW[8, d, {0, 0, 0, 2}] = If[(d ≥ 2), 70, 0];
nrSAW[8, d, {1, 0, 1, 1}] = If[(d ≥ 3), 280, 0];
nrSAW[8, d, {0, 2, 0, 1}] = If[(d ≥ 3), 420, 0];
nrSAW[8, d, {2, 1, 0, 1}] = If[(d ≥ 4), 840, 0];
nrSAW[8, d, {4, 0, 0, 1}] = If[(d ≥ 5), 1680, 0];
nrSAW[8, d, {0, 1, 2}] = If[(d ≥ 3), 560, 0];
nrSAW[8, d, {2, 0, 2}] = If[(d ≥ 4), 1120, 0];
nrSAW[8, d, {1, 2, 1}] = If[(d ≥ 4), 1680, 0];
nrSAW[8, d, {3, 1, 1}] = If[(d ≥ 5), 3360, 0];
nrSAW[8, d, {5, 0, 1}] = If[(d ≥ 6), 6720, 0];
nrSAW[8, d, {0, 4}] = If[(d ≥ 4), 2520, 0];

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nrSAW[8, d, {2, 3}] = If[(d ≥ 5), 5040, 0];
nrSAW[8, d, {4, 2}] = If[(d ≥ 6), 10 080, 0];
nrSAW[8, d, {6, 1}] = If[(d ≥ 7), 20 160, 0];
nrSAW[8, d, {8}] = If[(d ≥ 8), 40 320, 0];
(*n=8,BAW*)
nrBAW[8, d, {6}] = If[(d ≥ 6), 64 800 + 30 240 (d - 6), 0];
nrBAW[8, d, {4, 1}] = If[(d ≥ 5), 24 000 + 15 120 (d - 5), 0];
nrBAW[8, d, {4}] = If[(d ≥ 4), 19 896 + 36 048 (d - 4) + 11 520 (d - 4) (d - 5), 0];
nrBAW[8, d, {3, 0, 1}] = If[(d ≥ 4), 5760 + 5040 (d - 4), 0];
nrBAW[8, d, {2, 2}] = If[(d ≥ 4), 7800 + 7560 (d - 4), 0];
nrBAW[8, d, {2, 1}] = If[(d ≥ 3), 3318 + 11 544 (d - 3) + 5760 (d - 3) (d - 4), 0];
nrBAW[8, d, {2, 0, 0, 1}] = If[(d ≥ 3), 936 + 1260 (d - 3), 0];
nrBAW[8, d, {1, 1, 1}] = If[(d ≥ 3), 1480 + 2520 (d - 3), 0];
nrBAW[8, d, {0, 3}] = If[(d ≥ 3), 1800 + 3780 (d - 3), 0];
nrBAW[8, d, {2}] =
  If[(d ≥ 2), 164 + 4140 (d - 2) + 8248 (d - 2) (d - 3) + 2912 (d - 2) (d - 3) (d - 4), 0];
nrBAW[8, d, {1, 0, 1}] = If[(d ≥ 2), 162 + 2108 (d - 2) + 1920 (d - 2) (d - 3), 0];
nrBAW[8, d, {0, 2}] = If[(d ≥ 2), 184 + 2532 (d - 2) + 2880 (d - 2) (d - 3), 0];
nrBAW[8, d, {1, 0, 0, 0, 1}] = If[(d ≥ 2), 92 + 252 (d - 2), 0];
nrBAW[8, d, {0, 1, 0, 1}] = If[(d ≥ 2), 118 + 630 (d - 2), 0];
nrBAW[8, d, {0, 0, 2}] = If[(d ≥ 2), 120 + 840 (d - 2), 0];
nrBAW[8, d, {0, 1}] =
  If[(d ≥ 1), 188 (d - 1) + 1676 (d - 1) (d - 2) + 1456 (d - 1) (d - 2) (d - 3), 0];
nrBAW[8, d, {0, 0, 0, 1}] = If[(d ≥ 1), 134 (d - 1) + 480 (d - 1) (d - 2), 0];
nrBAW[8, d, {0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 42 (d - 1), 0];
nrBAW[8, d, {0}] = If[(d ≥ 0), 88 d (d - 1) + 624 d (d - 1) (d - 2) + 496 d (d - 1) (d - 2) (d - 3),
  0];
nrBAW[8, d, {0, 0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrBAW[8, d, {1, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 8, 0];
nrBAW[8, d, {0, 1, 0, 0, 0, 1}] = If[(d ≥ 2), 28, 0];
nrBAW[8, d, {2, 0, 0, 0, 0, 1}] = If[(d ≥ 3), 56, 0];
nrBAW[8, d, {0, 0, 1, 0, 1}] = If[(d ≥ 2), 56, 0];
nrBAW[8, d, {1, 1, 0, 0, 1}] = If[(d ≥ 3), 168, 0];
nrBAW[8, d, {3, 0, 0, 0, 1}] = If[(d ≥ 4), 336, 0];
nrBAW[8, d, {0, 0, 0, 2}] = If[(d ≥ 2), 70, 0];
nrBAW[8, d, {1, 0, 1, 1}] = If[(d ≥ 3), 280, 0];
nrBAW[8, d, {0, 2, 0, 1}] = If[(d ≥ 3), 420, 0];
nrBAW[8, d, {2, 1, 0, 1}] = If[(d ≥ 4), 840, 0];
nrBAW[8, d, {4, 0, 0, 1}] = If[(d ≥ 5), 1680, 0];
nrBAW[8, d, {0, 1, 2}] = If[(d ≥ 3), 560, 0];
nrBAW[8, d, {2, 0, 2}] = If[(d ≥ 4), 1120, 0];
nrBAW[8, d, {1, 2, 1}] = If[(d ≥ 4), 1680, 0];
nrBAW[8, d, {3, 1, 1}] = If[(d ≥ 5), 3360, 0];

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nrBAW[8, d, {5, 0, 1}] = If[(d ≥ 6), 6720, 0];
nrBAW[8, d, {0, 4}] = If[(d ≥ 4), 2520, 0];
nrBAW[8, d, {2, 3}] = If[(d ≥ 5), 5040, 0];
nrBAW[8, d, {4, 2}] = If[(d ≥ 6), 10 080, 0];
nrBAW[8, d, {6, 1}] = If[(d ≥ 7), 20 160, 0];
nrBAW[8, d, {8}] = If[(d ≥ 8), 40 320, 0];

(*n=9, SAW*)
nrSAW[9, d, {7}] = If[(d ≥ 7), 740 880 + 282 240 (d - 7), 0];
nrSAW[9, d, {5, 1}] = If[(d ≥ 6), 289 800 + 141 120 (d - 6), 0];
nrSAW[9, d, {5}] = If[(d ≥ 5), 305 520 + 424 800 (d - 5) + 108 000 (d - 5) (d - 6), 0];
nrSAW[9, d, {4, 0, 1}] = If[(d ≥ 5), 74 760 + 47 040 (d - 5), 0];
nrSAW[9, d, {3, 2}] = If[(d ≥ 5), 104 580 + 70 560 (d - 5), 0];
nrSAW[9, d, {3, 1}] = If[(d ≥ 4), 68 544 + 151 440 (d - 4) + 54 000 (d - 4) (d - 5), 0];
nrSAW[9, d, {3, 0, 0, 1}] = If[(d ≥ 4), 13 734 + 11 760 (d - 4), 0];
nrSAW[9, d, {2, 1, 1}] = If[(d ≥ 4), 23 940 + 23 520 (d - 4), 0];
nrSAW[9, d, {1, 3}] = If[(d ≥ 4), 32 130 + 35 280 (d - 4), 0];
nrSAW[9, d, {3}] =
  If[(d ≥ 3), 10 518 + 84 024 (d - 3) + 103 824 (d - 3) (d - 4) + 26 592 (d - 3) (d - 4) (d - 5), 0];
nrSAW[9, d, {2, 0, 1}] = If[(d ≥ 3), 8 146 + 34 080 (d - 3) + 18 000 (d - 3) (d - 4), 0];
nrSAW[9, d, {1, 2}] = If[(d ≥ 3), 9 204 + 45 240 (d - 3) + 27 000 (d - 3) (d - 4), 0];
nrSAW[9, d, {2, 0, 0, 0, 1}] = If[(d ≥ 3), 1 806 + 2 352 (d - 3), 0];
nrSAW[9, d, {1, 1, 0, 1}] = If[(d ≥ 3), 3 507 + 5 880 (d - 3), 0];
nrSAW[9, d, {1, 0, 2}] = If[(d ≥ 3), 4 340 + 7 840 (d - 3), 0];
nrSAW[9, d, {0, 2, 1}] = If[(d ≥ 3), 5 250 + 11 760 (d - 3), 0];
nrSAW[9, d, {1, 1}] =
  If[(d ≥ 2), 220 + 10 180 (d - 2) + 30 024 (d - 2) (d - 3) + 13 296 (d - 2) (d - 3) (d - 4), 0];
nrSAW[9, d, {1, 0, 0, 1}] = If[(d ≥ 2), 286 + 4 812 (d - 2) + 4 500 (d - 2) (d - 3), 0];
nrSAW[9, d, {0, 1, 1}] = If[(d ≥ 2), 277 + 6 880 (d - 2) + 9 000 (d - 2) (d - 3), 0];
nrSAW[9, d, {1, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 149 + 392 (d - 2), 0];
nrSAW[9, d, {0, 1, 0, 0, 1}] = If[(d ≥ 2), 231 + 1 176 (d - 2), 0];
nrSAW[9, d, {0, 0, 1, 1}] = If[(d ≥ 2), 259 + 1 960 (d - 2), 0];
nrSAW[9, d, {1}] =
  If[(d ≥ 1), 140 (d - 1) + 3 880 (d - 1) (d - 2) + 9 640 (d - 1) (d - 2) (d - 3) +
    3 968 (d - 1) (d - 2) (d - 3) (d - 4), 0];
nrSAW[9, d, {0, 0, 1}] =
  If[(d ≥ 3), 256 (d - 1) + 4 212 (d - 1) (d - 2) + 4 432 (d - 1) (d - 2) (d - 3), 0];
nrSAW[9, d, {0, 0, 0, 0, 1}] = If[(d ≥ 2), 260 (d - 1) + 900 (d - 1) (d - 2), 0];
nrSAW[9, d, {0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 56 (d - 1), 0];
nrSAW[9, d, {1, 0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 9, 0];
nrSAW[9, d, {0, 1, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 36, 0];
nrSAW[9, d, {2, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 3), 72, 0];
nrSAW[9, d, {0, 0, 1, 0, 0, 1}] = If[(d ≥ 2), 84, 0];

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nrSAW[9, d, {1, 1, 0, 0, 0, 1}] = If[(d ≥ 3), 252, 0];
nrSAW[9, d, {3, 0, 0, 0, 0, 1}] = If[(d ≥ 4), 504, 0];
nrSAW[9, d, {0, 0, 0, 1, 1}] = If[(d ≥ 2), 126, 0];
nrSAW[9, d, {1, 0, 1, 0, 1}] = If[(d ≥ 3), 504, 0];
nrSAW[9, d, {0, 2, 0, 0, 1}] = If[(d ≥ 3), 756, 0];
nrSAW[9, d, {2, 1, 0, 0, 1}] = If[(d ≥ 4), 1512, 0];
nrSAW[9, d, {4, 0, 0, 0, 1}] = If[(d ≥ 5), 3024, 0];
nrSAW[9, d, {1, 0, 0, 2}] = If[(d ≥ 3), 630, 0];
nrSAW[9, d, {0, 1, 1, 1}] = If[(d ≥ 3), 1260, 0];
nrSAW[9, d, {2, 0, 1, 1}] = If[(d ≥ 4), 2520, 0];
nrSAW[9, d, {1, 2, 0, 1}] = If[(d ≥ 4), 3780, 0];
nrSAW[9, d, {3, 1, 0, 1}] = If[(d ≥ 5), 7560, 0];
nrSAW[9, d, {5, 0, 0, 1}] = If[(d ≥ 6), 15120, 0];
nrSAW[9, d, {0, 0, 3}] = If[(d ≥ 3), 1680, 0];
nrSAW[9, d, {1, 1, 2}] = If[(d ≥ 4), 5040, 0];
nrSAW[9, d, {3, 0, 2}] = If[(d ≥ 5), 10080, 0];
nrSAW[9, d, {0, 3, 1}] = If[(d ≥ 4), 7560, 0];
nrSAW[9, d, {2, 2, 1}] = If[(d ≥ 5), 15120, 0];
nrSAW[9, d, {4, 1, 1}] = If[(d ≥ 6), 30240, 0];
nrSAW[9, d, {6, 0, 1}] = If[(d ≥ 7), 60480, 0];
nrSAW[9, d, {1, 4}] = If[(d ≥ 5), 22680, 0];
nrSAW[9, d, {3, 3}] = If[(d ≥ 6), 45360, 0];
nrSAW[9, d, {5, 2}] = If[(d ≥ 7), 90720, 0];
nrSAW[9, d, {7, 1}] = If[(d ≥ 8), 181440, 0];
nrSAW[9, d, {9}] = If[(d ≥ 9), 362880, 0];

(*n=9, BAW*)
nrBAW[9, d, {7}] = If[(d ≥ 7), 740880 + 282240 (d - 7), 0];
nrBAW[9, d, {5, 1}] = If[(d ≥ 6), 289800 + 141120 (d - 6), 0];
nrBAW[9, d, {5}] = If[(d ≥ 5), 344880 + 448800 (d - 5) + 110880 (d - 5) (d - 6), 0];
nrBAW[9, d, {4, 0, 1}] = If[(d ≥ 5), 74760 + 47040 (d - 5), 0];
nrBAW[9, d, {3, 2}] = If[(d ≥ 5), 104580 + 70560 (d - 5), 0];
nrBAW[9, d, {3, 1}] = If[(d ≥ 4), 79056 + 160560 (d - 4) + 55440 (d - 4) (d - 5), 0];
nrBAW[9, d, {3, 0, 0, 1}] = If[(d ≥ 4), 13734 + 11760 (d - 4), 0];
nrBAW[9, d, {2, 1, 1}] = If[(d ≥ 4), 23940 + 23520 (d - 4), 0];
nrBAW[9, d, {1, 3}] = If[(d ≥ 4), 32130 + 35280 (d - 4), 0];
nrBAW[9, d, {3}] =
  If[(d ≥ 3), 17118 + 111996 (d - 3) + 121896 (d - 3) (d - 4) + 9088 (d - 3) (d - 4) (d - 5), 0];
nrBAW[9, d, {2, 0, 1}] = If[(d ≥ 3), 9538 + 36160 (d - 3) + 18480 (d - 3) (d - 4), 0];
nrBAW[9, d, {1, 2}] = If[(d ≥ 3), 11316 + 48360 (d - 3) + 27720 (d - 3) (d - 4), 0];
nrBAW[9, d, {2, 0, 0, 0, 1}] = If[(d ≥ 3), 1806 + 2352 (d - 3), 0];
nrBAW[9, d, {1, 1, 0, 1}] = If[(d ≥ 3), 3507 + 5880 (d - 3), 0];
nrBAW[9, d, {1, 0, 2}] = If[(d ≥ 3), 4340 + 7840 (d - 3), 0];

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nrBAW[9, d, {0, 2, 1}] = If[(d ≥ 3), 5250 + 11760 (d - 3), 0];
nrBAW[9, d, {1, 1}] =
  If[(d ≥ 2), 432 + 14662 (d - 2) + 35796 (d - 2) (d - 3) + 14544 (d - 2) (d - 3) (d - 4), 0];
nrBAW[9, d, {1, 0, 0, 1}] = If[(d ≥ 2), 342 + 5092 (d - 2) + 4620 (d - 2) (d - 3), 0];
nrBAW[9, d, {0, 1, 1}] = If[(d ≥ 2), 405 + 7440 (d - 2) + 9240 (d - 2) (d - 3), 0];
nrBAW[9, d, {1, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 149 + 392 (d - 2), 0];
nrBAW[9, d, {0, 1, 0, 0, 1}] = If[(d ≥ 2), 231 + 1176 (d - 2), 0];
nrBAW[9, d, {0, 0, 1, 1}] = If[(d ≥ 2), 259 + 1960 (d - 2), 0];
nrBAW[9, d, {1}] =
  If[(d ≥ 1), 408 (d - 1) + 8656 (d - 1) (d - 2) + 16712 (d - 1) (d - 2) (d - 3) +
    5616 (d - 1) (d - 2) (d - 3) (d - 4), 0];
nrBAW[9, d, {0, 0, 1}] =
  If[(d ≥ 1), 438 (d - 1) + 5088 (d - 1) (d - 2) + 4848 (d - 1) (d - 2) (d - 3), 0];
nrBAW[9, d, {0, 0, 0, 0, 1}] = If[(d ≥ 1), 268 (d - 1) + 924 (d - 1) (d - 2), 0];
nrBAW[9, d, {0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 56 (d - 1), 0];
nrBAW[9, d, {1, 0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 9, 0];
nrBAW[9, d, {0, 1, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 36, 0];
nrBAW[9, d, {2, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 3), 72, 0];
nrBAW[9, d, {0, 0, 1, 0, 0, 1}] = If[(d ≥ 3), 84, 0];
nrBAW[9, d, {1, 1, 0, 0, 0, 1}] = If[(d ≥ 3), 252, 0];
nrBAW[9, d, {3, 0, 0, 0, 0, 1}] = If[(d ≥ 4), 504, 0];
nrBAW[9, d, {0, 0, 0, 1, 1}] = If[(d ≥ 2), 126, 0];
nrBAW[9, d, {1, 0, 1, 0, 1}] = If[(d ≥ 3), 504, 0];
nrBAW[9, d, {0, 2, 0, 0, 1}] = If[(d ≥ 3), 756, 0];
nrBAW[9, d, {2, 1, 0, 0, 1}] = If[(d ≥ 4), 1512, 0];
nrBAW[9, d, {4, 0, 0, 0, 1}] = If[(d ≥ 5), 3024, 0];
nrBAW[9, d, {1, 0, 0, 2}] = If[(d ≥ 3), 630, 0];
nrBAW[9, d, {0, 1, 1, 1}] = If[(d ≥ 3), 1260, 0];
nrBAW[9, d, {2, 0, 1, 1}] = If[(d ≥ 4), 2520, 0];
nrBAW[9, d, {1, 2, 0, 1}] = If[(d ≥ 4), 3780, 0];
nrBAW[9, d, {3, 1, 0, 1}] = If[(d ≥ 5), 7560, 0];
nrBAW[9, d, {5, 0, 0, 1}] = If[(d ≥ 6), 15120, 0];
nrBAW[9, d, {0, 0, 3}] = If[(d ≥ 3), 1680, 0];
nrBAW[9, d, {1, 1, 2}] = If[(d ≥ 4), 5040, 0];
nrBAW[9, d, {3, 0, 2}] = If[(d ≥ 5), 10080, 0];
nrBAW[9, d, {0, 3, 1}] = If[(d ≥ 4), 7560, 0];
nrBAW[9, d, {2, 2, 1}] = If[(d ≥ 5), 15120, 0];
nrBAW[9, d, {4, 1, 1}] = If[(d ≥ 6), 30240, 0];
nrBAW[9, d, {6, 0, 1}] = If[(d ≥ 7), 60480, 0];
nrBAW[9, d, {1, 4}] = If[(d ≥ 5), 22680, 0];
nrBAW[9, d, {3, 3}] = If[(d ≥ 6), 45360, 0];
nrBAW[9, d, {5, 2}] = If[(d ≥ 7), 90720, 0];
nrBAW[9, d, {7, 1}] = If[(d ≥ 8), 181440, 0];

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(*n=10, SAW*)

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nrSAW[10, d, {8}] = If[(d ≥ 8), 9031680 + 2903040 (d - 8), 0];
nrSAW[10, d, {6, 1}] = If[(d ≥ 7), 3669120 + 1451520 (d - 7), 0];
nrSAW[10, d, {6}] = If[(d ≥ 6), 5342400 + 5620320 (d - 6) + 1149120 (d - 6) (d - 7), 0];
nrSAW[10, d, {5, 0, 1}] = If[(d ≥ 6), 991200 + 483840 (d - 6), 0];
nrSAW[10, d, {4, 2}] = If[(d ≥ 6), 1411200 + 725760 (d - 6), 0];
nrSAW[10, d, {4, 1}] = If[(d ≥ 5), 1442040 + 2144880 (d - 5) + 574560 (d - 5) (d - 6), 0];
nrSAW[10, d, {4, 0, 0, 1}] = If[(d ≥ 5), 194880 + 120960 (d - 5), 0];
nrSAW[10, d, {3, 1, 1}] = If[(d ≥ 5), 354480 + 241920 (d - 5), 0];
nrSAW[10, d, {2, 3}] = If[(d ≥ 5), 493920 + 362880 (d - 5), 0];
nrSAW[10, d, {4}] =
  If[(d ≥ 4), 508608 + 1990944 (d - 4) + 1585920 (d - 4) (d - 5) + 297600 (d - 4) (d - 5) (d - 6),
    0];
nrSAW[10, d, {3, 0, 1}] = If[(d ≥ 4), 236760 + 533520 (d - 4) + 191520 (d - 4) (d - 5), 0];
nrSAW[10, d, {2, 2}] = If[(d ≥ 4), 297960 + 739800 (d - 4) + 287280 (d - 4) (d - 5), 0];
nrSAW[10, d, {3, 0, 0, 0, 1}] = If[(d ≥ 4), 28896 + 24192 (d - 4), 0];
nrSAW[10, d, {2, 1, 0, 1}] = If[(d ≥ 4), 62160 + 60480 (d - 4), 0];
nrSAW[10, d, {2, 0, 2}] = If[(d ≥ 4), 79520 + 80640 (d - 4), 0];
nrSAW[10, d, {1, 2, 1}] = If[(d ≥ 4), 106680 + 120960 (d - 4), 0];
nrSAW[10, d, {0, 4}] = If[(d ≥ 4), 141120 + 181440 (d - 4), 0];
nrSAW[10, d, {2, 1}] =
  If[(d ≥ 3), 41524 + 392436 (d - 3) + 538080 (d - 3) (d - 4) + 148800 (d - 3) (d - 4) (d - 5), 0];
nrSAW[10, d, {2, 0, 0, 1}] = If[(d ≥ 3), 22104 + 92052 (d - 3) + 47880 (d - 3) (d - 4), 0];
nrSAW[10, d, {1, 1, 1}] = If[(d ≥ 3), 29440 + 155880 (d - 3) + 95760 (d - 3) (d - 4), 0];
nrSAW[10, d, {0, 3}] = If[(d ≥ 3), 33210 + 203580 (d - 3) + 143640 (d - 3) (d - 4), 0];
nrSAW[10, d, {2, 0, 0, 0, 0, 1}] = If[(d ≥ 3), 3184 + 4032 (d - 3), 0];
nrSAW[10, d, {1, 1, 0, 0, 1}] = If[(d ≥ 3), 7392 + 12096 (d - 3), 0];
nrSAW[10, d, {1, 0, 1, 1}] = If[(d ≥ 3), 11060 + 20160 (d - 3), 0];
nrSAW[10, d, {0, 2, 0, 1}] = If[(d ≥ 3), 13440 + 30240 (d - 3), 0];
nrSAW[10, d, {0, 1, 2}] = If[(d ≥ 3), 16240 + 40320 (d - 3), 0];
nrSAW[10, d, {2}] =
  If[(d ≥ 2), 396 + 38580 (d - 2) + 223560 (d - 2) (d - 3) + 236928 (d - 2) (d - 3) (d - 4) +
    54336 (d - 2) (d - 3) (d - 4) (d - 5), 0];
nrSAW[10, d, {1, 0, 1}] =
  If[(d ≥ 2), 598 + 35344 (d - 2) + 110360 (d - 2) (d - 3) + 49600 (d - 2) (d - 3) (d - 4), 0];
nrSAW[10, d, {0, 2}] =
  If[(d ≥ 2), 568 + 38700 (d - 2) + 141600 (d - 2) (d - 3) + 74400 (d - 2) (d - 3) (d - 4), 0];
nrSAW[10, d, {1, 0, 0, 0, 1}] = If[(d ≥ 2), 636 + 10548 (d - 2) + 9576 (d - 2) (d - 3), 0];
nrSAW[10, d, {0, 1, 0, 1}] = If[(d ≥ 2), 661 + 18306 (d - 2) + 23940 (d - 2) (d - 3), 0];
nrSAW[10, d, {0, 0, 2}] = If[(d ≥ 2), 660 + 21720 (d - 2) + 31920 (d - 2) (d - 3), 0];
nrSAW[10, d, {1, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 226 + 576 (d - 2), 0];
nrSAW[10, d, {0, 1, 0, 0, 0, 1}] = If[(d ≥ 2), 416 + 2016 (d - 2), 0];
nrSAW[10, d, {0, 0, 1, 0, 1}] = If[(d ≥ 2), 532 + 4032 (d - 2), 0];
nrSAW[10, d, {0, 0, 0, 2}] = If[(d ≥ 2), 560 + 5040 (d - 2), 0];

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nrSAW[10, d, {0, 1}] =
  If[(d ≥ 1), 468 (d - 1) + 19904 (d - 1) (d - 2) + 59392 (d - 1) (d - 2) (d - 3) +
    27168 (d - 1) (d - 2) (d - 3) (d - 4), 0];
nrSAW[10, d, {0, 0, 0, 1}] =
  If[(d ≥ 1), 654 (d - 1) + 11936 (d - 1) (d - 2) + 12400 (d - 1) (d - 2) (d - 3), 0];
nrSAW[10, d, {0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 486 (d - 1) + 1596 (d - 1) (d - 2), 0];
nrSAW[10, d, {0, 0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 72 (d - 1), 0];
nrBAW[10, d, {0, 0, 0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrSAW[10, d, {1, 0, 0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 10, 0];
nrSAW[10, d, {0, 1, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 45, 0];
nrSAW[10, d, {2, 0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 3), 90, 0];
nrSAW[10, d, {0, 0, 1, 0, 0, 0, 1}] = If[(d ≥ 2), 120, 0];
nrSAW[10, d, {1, 1, 0, 0, 0, 0, 1}] = If[(d ≥ 3), 360, 0];
nrSAW[10, d, {3, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 4), 720, 0];
nrSAW[10, d, {0, 0, 0, 1, 0, 1}] = If[(d ≥ 2), 210, 0];
nrSAW[10, d, {1, 0, 1, 0, 0, 1}] = If[(d ≥ 3), 840, 0];
nrSAW[10, d, {0, 2, 0, 0, 0, 1}] = If[(d ≥ 3), 1260, 0];
nrSAW[10, d, {2, 1, 0, 0, 0, 1}] = If[(d ≥ 4), 2520, 0];
nrSAW[10, d, {4, 0, 0, 0, 0, 1}] = If[(d ≥ 5), 5040, 0];
nrSAW[10, d, {0, 0, 0, 0, 2}] = If[(d ≥ 2), 252, 0];
nrSAW[10, d, {1, 0, 0, 1, 1}] = If[(d ≥ 3), 1260, 0];
nrSAW[10, d, {0, 1, 1, 0, 1}] = If[(d ≥ 3), 2520, 0];
nrSAW[10, d, {2, 0, 1, 0, 1}] = If[(d ≥ 4), 5040, 0];
nrSAW[10, d, {1, 2, 0, 0, 1}] = If[(d ≥ 4), 7560, 0];
nrSAW[10, d, {3, 1, 0, 0, 1}] = If[(d ≥ 5), 15120, 0];
nrSAW[10, d, {5, 0, 0, 0, 1}] = If[(d ≥ 6), 30240, 0];
nrSAW[10, d, {0, 1, 0, 2}] = If[(d ≥ 3), 3150, 0];
nrSAW[10, d, {2, 0, 0, 2}] = If[(d ≥ 4), 6300, 0];
nrSAW[10, d, {0, 0, 2, 1}] = If[(d ≥ 3), 4200, 0];
nrSAW[10, d, {1, 1, 1, 1}] = If[(d ≥ 4), 12600, 0];
nrSAW[10, d, {3, 0, 1, 1}] = If[(d ≥ 5), 25200, 0];
nrSAW[10, d, {0, 3, 0, 1}] = If[(d ≥ 4), 18900, 0];
nrSAW[10, d, {2, 2, 0, 1}] = If[(d ≥ 5), 37800, 0];
nrSAW[10, d, {4, 1, 0, 1}] = If[(d ≥ 6), 75600, 0];
nrSAW[10, d, {6, 0, 0, 1}] = If[(d ≥ 7), 151200, 0];
nrSAW[10, d, {1, 0, 3}] = If[(d ≥ 4), 16800, 0];
nrSAW[10, d, {0, 2, 2}] = If[(d ≥ 4), 25200, 0];
nrSAW[10, d, {2, 1, 2}] = If[(d ≥ 5), 50400, 0];
nrSAW[10, d, {4, 0, 2}] = If[(d ≥ 6), 100800, 0];
nrSAW[10, d, {1, 3, 1}] = If[(d ≥ 5), 75600, 0];
nrSAW[10, d, {3, 2, 1}] = If[(d ≥ 6), 151200, 0];
nrSAW[10, d, {5, 1, 1}] = If[(d ≥ 7), 302400, 0];
nrSAW[10, d, {7, 0, 1}] = If[(d ≥ 8), 604800, 0];

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nrSAW[10, d, {0, 5}] = If[(d ≥ 5), 113400, 0];
nrSAW[10, d, {2, 4}] = If[(d ≥ 6), 226800, 0];
nrSAW[10, d, {4, 3}] = If[(d ≥ 7), 453600, 0];
nrSAW[10, d, {6, 2}] = If[(d ≥ 8), 907200, 0];
nrSAW[10, d, {8, 1}] = If[(d ≥ 9), 1814400, 0];
nrSAW[10, d, {10}] = If[(d ≥ 10), 3628800, 0];
(*n=10,BAW*)
nrBAW[10, d, {8}] = If[(d ≥ 8), 9031680 + 2903040 (d - 8), 0];
nrBAW[10, d, {6, 1}] = If[(d ≥ 7), 3669120 + 1451520 (d - 7), 0];
nrBAW[10, d, {6}] = If[(d ≥ 6), 5781600 + 5827680 (d - 6) + 1169280 (d - 6) (d - 7), 0];
nrBAW[10, d, {5, 0, 1}] = If[(d ≥ 6), 991200 + 483840 (d - 6), 0];
nrBAW[10, d, {4, 2}] = If[(d ≥ 6), 1411200 + 725760 (d - 6), 0];
nrBAW[10, d, {4, 1}] = If[(d ≥ 5), 1577880 + 2228400 (d - 5) + 584640 (d - 5) (d - 6), 0];
nrBAW[10, d, {4, 0, 0, 1}] = If[(d ≥ 5), 194880 + 120960 (d - 5), 0];
nrBAW[10, d, {3, 1, 1}] = If[(d ≥ 5), 354480 + 241920 (d - 5), 0];
nrBAW[10, d, {2, 3}] = If[(d ≥ 5), 493920 + 362880 (d - 5), 0];
nrBAW[10, d, {4}] =
  If[(d ≥ 4), 683904 + 2377728 (d - 4) + 1753632 (d - 4) (d - 5) + 314880 (d - 4) (d - 5) (d - 6),
    0];
nrBAW[10, d, {3, 0, 1}] = If[(d ≥ 4), 260760 + 554640 (d - 4) + 194880 (d - 4) (d - 5), 0];
nrBAW[10, d, {2, 2}] = If[(d ≥ 4), 334080 + 771480 (d - 4) + 292320 (d - 4) (d - 5), 0];
nrBAW[10, d, {3, 0, 0, 0, 1}] = If[(d ≥ 4), 28896 + 24192 (d - 4), 0];
nrBAW[10, d, {2, 1, 0, 1}] = If[(d ≥ 4), 62160 + 60480 (d - 4), 0];
nrBAW[10, d, {2, 0, 2}] = If[(d ≥ 4), 79520 + 80640 (d - 4), 0];
nrBAW[10, d, {1, 2, 1}] = If[(d ≥ 4), 106680 + 120960 (d - 4), 0];
nrBAW[10, d, {0, 4}] = If[(d ≥ 4), 141120 + 181440 (d - 4), 0];
nrBAW[10, d, {2, 1}] =
  If[(d ≥ 3), 61380 + 484164 (d - 3) + 599376 (d - 3) (d - 4) + 157440 (d - 3) (d - 4) (d - 5), 0];
nrBAW[10, d, {2, 0, 0, 1}] = If[(d ≥ 3), 24444 + 95652 (d - 3) + 48720 (d - 3) (d - 4), 0];
nrBAW[10, d, {1, 1, 1}] = If[(d ≥ 3), 34200 + 163080 (d - 3) + 97440 (d - 3) (d - 4), 0];
nrBAW[10, d, {0, 3}] = If[(d ≥ 3), 40410 + 214380 (d - 3) + 146160 (d - 3) (d - 4), 0];
nrBAW[10, d, {2, 0, 0, 0, 0, 1}] = If[(d ≥ 3), 3184 + 4032 (d - 3), 0];
nrBAW[10, d, {1, 1, 0, 0, 1}] = If[(d ≥ 3), 7392 + 12096 (d - 3), 0];
nrBAW[10, d, {1, 0, 1, 1}] = If[(d ≥ 3), 11060 + 20160 (d - 3), 0];
nrBAW[10, d, {0, 2, 0, 1}] = If[(d ≥ 3), 13440 + 30240 (d - 3), 0];
nrBAW[10, d, {0, 1, 2}] = If[(d ≥ 3), 16240 + 40320 (d - 3), 0];
nrBAW[10, d, {2}] =
  If[(d ≥ 2), 1052 + 74060 (d - 2) + 339144 (d - 2) (d - 3) + 305520 (d - 2) (d - 3) (d - 4) +
    63360 (d - 2) (d - 3) (d - 4) (d - 5), 0];
nrBAW[10, d, {1, 0, 1}] =
  If[(d ≥ 2), 1070 + 45408 (d - 2) + 123552 (d - 2) (d - 3) + 52480 (d - 2) (d - 3) (d - 4), 0];
nrBAW[10, d, {0, 2}] =
  If[(d ≥ 2), 1048 + 52932 (d - 2) + 160968 (d - 2) (d - 3) + 78720 (d - 2) (d - 3) (d - 4), 0];

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nrBAW[10, d, {1, 0, 0, 0, 1}] = If[(d ≥ 2), 704 + 10932 (d - 2) + 9744 (d - 2) (d - 3), 0];
nrBAW[10, d, {0, 1, 0, 1}] = If[(d ≥ 2), 861 + 19266 (d - 2) + 24360 (d - 2) (d - 3), 0];
nrBAW[10, d, {0, 0, 2}] = If[(d ≥ 2), 940 + 23000 (d - 2) + 32480 (d - 2) (d - 3), 0];
nrBAW[10, d, {1, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 226 + 576 (d - 2), 0];
nrBAW[10, d, {0, 1, 0, 0, 0, 1}] = If[(d ≥ 2), 416 + 2016 (d - 2), 0];
nrBAW[10, d, {0, 0, 1, 0, 1}] = If[(d ≥ 2), 532 + 4032 (d - 2), 0];
nrBAW[10, d, {0, 0, 0, 2}] = If[(d ≥ 2), 560 + 5040 (d - 2), 0];
nrBAW[10, d, {0, 1}] =
  If[(d ≥ 1), 1042 (d - 1) + 33076 (d - 1) (d - 2) + 78520 (d - 1) (d - 2) (d - 3) +
    31680 (d - 1) (d - 2) (d - 3) (d - 4), 0];
nrBAW[10, d, {0, 0, 0, 1}] =
  If[(d ≥ 1), 954 (d - 1) + 13452 (d - 1) (d - 2) + 13120 (d - 1) (d - 2) (d - 3), 0];
nrBAW[10, d, {0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 494 (d - 1) + 1624 (d - 1) (d - 2), 0];
nrBAW[10, d, {0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 72 (d - 1), 0];
nrBAW[10, d, {0}] =
  If[(d ≥ 0), 440 d (d - 1) + 11680 d (d - 1) (d - 2) + 24720 d (d - 1) (d - 2) (d - 3) +
    9216 d (d - 1) (d - 2) (d - 3) (d - 4), 0];
nrBAW[10, d, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 1), 1, 0];
nrBAW[10, d, {1, 0, 0, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 10, 0];
nrBAW[10, d, {0, 1, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 2), 45, 0];
nrBAW[10, d, {2, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 3), 90, 0];
nrBAW[10, d, {0, 0, 1, 0, 0, 0, 1}] = If[(d ≥ 2), 120, 0];
nrBAW[10, d, {1, 1, 0, 0, 0, 0, 1}] = If[(d ≥ 3), 360, 0];
nrBAW[10, d, {3, 0, 0, 0, 0, 0, 1}] = If[(d ≥ 4), 720, 0];
nrBAW[10, d, {0, 0, 0, 1, 0, 1}] = If[(d ≥ 2), 210, 0];
nrBAW[10, d, {1, 0, 1, 0, 0, 1}] = If[(d ≥ 3), 840, 0];
nrBAW[10, d, {0, 2, 0, 0, 0, 1}] = If[(d ≥ 3), 1260, 0];
nrBAW[10, d, {2, 1, 0, 0, 0, 1}] = If[(d ≥ 4), 2520, 0];
nrBAW[10, d, {4, 0, 0, 0, 0, 1}] = If[(d ≥ 5), 5040, 0];
nrBAW[10, d, {0, 0, 0, 0, 2}] = If[(d ≥ 2), 252, 0];
nrBAW[10, d, {1, 0, 0, 1, 1}] = If[(d ≥ 3), 1260, 0];
nrBAW[10, d, {0, 1, 1, 0, 1}] = If[(d ≥ 3), 2520, 0];
nrBAW[10, d, {2, 0, 1, 0, 1}] = If[(d ≥ 4), 5040, 0];
nrBAW[10, d, {1, 2, 0, 0, 1}] = If[(d ≥ 4), 7560, 0];
nrBAW[10, d, {3, 1, 0, 0, 1}] = If[(d ≥ 5), 15120, 0];
nrBAW[10, d, {5, 0, 0, 0, 1}] = If[(d ≥ 6), 30240, 0];
nrBAW[10, d, {0, 1, 0, 2}] = If[(d ≥ 3), 3150, 0];
nrBAW[10, d, {2, 0, 0, 2}] = If[(d ≥ 4), 6300, 0];
nrBAW[10, d, {0, 0, 2, 1}] = If[(d ≥ 3), 4200, 0];
nrBAW[10, d, {1, 1, 1, 1}] = If[(d ≥ 4), 12600, 0];
nrBAW[10, d, {3, 0, 1, 1}] = If[(d ≥ 5), 25200, 0];
nrBAW[10, d, {0, 3, 0, 1}] = If[(d ≥ 4), 18900, 0];
nrBAW[10, d, {2, 2, 0, 1}] = If[(d ≥ 5), 37800, 0];

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nrBAW[10, d, {4, 1, 0, 1}] = If[(d ≥ 6), 75 600, 0];  
nrBAW[10, d, {6, 0, 0, 1}] = If[(d ≥ 7), 151 200, 0];  
nrBAW[10, d, {1, 0, 3}] = If[(d ≥ 4), 16 800, 0];  
nrBAW[10, d, {0, 2, 2}] = If[(d ≥ 4), 25 200, 0];  
nrBAW[10, d, {2, 1, 2}] = If[(d ≥ 5), 50 400, 0];  
nrBAW[10, d, {4, 0, 2}] = If[(d ≥ 6), 100 800, 0];  
nrBAW[10, d, {1, 3, 1}] = If[(d ≥ 5), 75 600, 0];  
nrBAW[10, d, {3, 2, 1}] = If[(d ≥ 6), 151 200, 0];  
nrBAW[10, d, {5, 1, 1}] = If[(d ≥ 7), 302 400, 0];  
nrBAW[10, d, {7, 0, 1}] = If[(d ≥ 8), 604 800, 0];  
nrBAW[10, d, {0, 5}] = If[(d ≥ 5), 113 400, 0];  
nrBAW[10, d, {2, 4}] = If[(d ≥ 6), 226 800, 0];  
nrBAW[10, d, {4, 3}] = If[(d ≥ 7), 453 600, 0];  
nrBAW[10, d, {6, 2}] = If[(d ≥ 8), 907 200, 0];  
nrBAW[10, d, {8, 1}] = If[(d ≥ 9), 1 814 400, 0];  
nrBAW[10, d, {10}] = If[(d ≥ 10), 3 628 800, 0];
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